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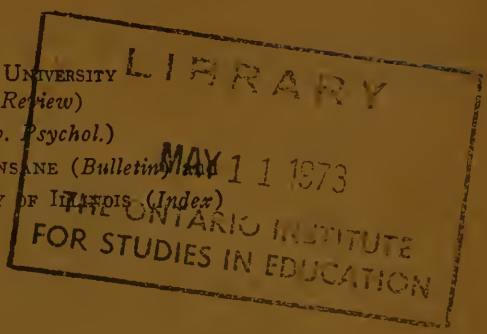

The Interrelation of Some Higher Learning Processes

BY

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FOREWORD

The writer is greatly indebted to Dr. Joseph Peterson whose guidance and personal interest have been of inestimable value throughout the conduct of this investigation and whose scientific attitude has been a source of great inspiration. He also wishes to take this opportunity to express his thanks and appreciation to the students who served patiently as subjects to make the investigation possible, and especially to his wife who assisted greatly in making the mathematical calculations.

ERRATA

Attention is directed to the following list of corrections of errors, which unfortunately were not remarked in time to be set right in the body of the text:

In Table VIII, p. 21, "100-19," lower left corner, should read "0-19." "50-64" in top row should read "60-64."

In Table X, p. 23, "0-10" in top row should read "0-19."

In Table XV, p. 31, the omitted spaces in the base line should be 6 and 8, respectively, from left to right; and "20-25" in the top row should read "20-24."

In Table XVI, p. 32, "(Modified)" should be added at the end of the line just over the figure, beginning "X = ."

In Table XVII, p. 33, numbers omitted in lower row should be, from left to right: 3, 2, 3, 5, 7, 8, 9, 9, 7, 6, 7, 3, 3, and 2.

In Table XXII, p. 41, "54-60" in first column should read "64-60;" and "35-35" in the top row should read "35-39."

In Table XXV, p. 47, 7 and 17 in the 9th and the 10th squares of the bottom row should be 11 and 13, respectively.

TABLE OF CONTENTS

THE INTERRELATION OF SOME HIGHER LEARNING PROCESSES

I. Problem

The purpose of this investigation is to analyze and to study the interrelations of some higher mental processes. Each of the experiments used involves two or more factors,[1] such as time, repetitions, solutions, errors, etc. Each problem requires considerable time for the learning and in this respect is different from the individual parts of most intelligence tests. The problems are also of such a nature that the subject may solve them either by the hit-and-miss method or by a reflective method. In every case it is possible to record all the responses of the subject, and thus provide objective data for the analysis.

The purpose is to compare each factor in the tests with a criterion and with every other factor. The tests will be scored by combining the significant factors and they will then be further compared with the criterion and with each other. In this way it is believed that a detailed analysis can be made that will throw light on methods of learning and on the characteristics of tests involving the higher mental processes. This procedure will analyze not only the tests used, but also the criterion. An attempt will be made to answer such questions as the following: Does time measure any elements in learning that are not measured by the criterion? Does time measure any elements in learning that are not measured by repetitions or errors? Does one rational learning experiment involve the same functions as any other? Is there a general rational learning function, or are such types of learning simply operations of various factors in different sorts of combinations.

[1] Factor is used in this investigation to designate one kind of data, such as time, repetitions, solutions, errors, etc.

II. Method

The raw scores in each experiment are first put into percentiles by use of Rugg's table.[2] In order to shorten the work a table based on seventy-four cases, the number used in this investigation, was constructed. Three steps are involved in making such a table. First, the numbers from 1 to 74 are divided by 74, giving the percent of subjects making each score. Second, since Rugg's table provides for the percent failing, it is necessary to subtract each of these percents from 100, getting the percent below each score. Third, the percentile[3] corresponding to each number obtained in the second step is taken from the table. An illustration will serve to make the method clearer. We shall take the subject making the highest score. He ranks number 1. This number divided by 74 gives 1.35 percent. If we subtract 1.35 from 100, we get 98.65, the percent of subjects below the best one. The percentile in the table corresponding to 98.65 is 86. Then 86 is the percentile rank of the subject having the highest rank in any test or factor of a test. The corresponding percentiles are found in this manner for each rank and then it is necessary only to rank the subjects by the usual method and read off the percentiles from the table.

There may be a slight objection to this method of assigning percentiles. The question may well be asked as to why the scores run down to 0 and yet up only to 86. Why should they not go up to 100? By this method the upper score will approach 100 as the number of cases is increased. If we think of the scale as a continuous one, we may regard 0 as extending from 0 to 14 and 86 as extending from 86 to 100. It would probably have been a little more correct to have moved each score up a half step or to have designated its middle position in a continuous

[2] Rugg, H. O., *Statistical Method Applied to Education*, 1917, 396 ff.

[3] Scores are assumed to fit the probability curve and percentages of subjects who make various scores correspond to percentages of area under the curve from the 0 point to a point on the base line. This point on base line is measured in units of σ and is transformed into percentile scores by setting 0 at $-3.0\ \sigma$, 50 at the mean and 100 at $+ 3.0\ \sigma$.

Example: A subject having 20 per cent of the subjects below him will always have a percentile score of 36.

scale. This would have put the lowest scale at 7 and the highest at 93. Certainly there would have been no difference in the results as far as correlations are concerned. It would have thrown the mean in each distribution just a small fraction of a unit higher and would not have changed the standard deviations.

The reason for not using the raw scores probably needs no defense here. The percentiles as used here tend to give a more nearly normal distribution than that given by the raw scores. Nearly all devices for handling and refining data are based on normal distribution. Probably scores that give a normal distribution are more nearly correct than raw scores. Who knows whether five minutes consumed at the beginning of the experiment is just as significant as five minutes after the subject has been working thirty minutes? Again, is there any evidence that two hundred errors mean just half as much efficiency as one hundred? Very often one little confusion or distraction will cause the number of errors to be doubled. A second reason may be given for using the percentiles. All standard deviation units are, for all practical purposes, equal[4] and thus simplify analysis by the use of regression lines. The regression of x on y will always be equal to y on x when the standard deviations of the two arrays are equal. Also in the distribution tables the number of rows will be equal to the number of columns and thus make the regression lines less likely to be misinterpreted.

After the data have all been reduced to percentiles, the next problem is to determine the method of scoring or combining the different factors in the test. This method of scoring is secondary to that of analysis. Such questions as the following must be answered: Is it necessary to use all the factors in the tests? If not, what ones should be used? The answers to such questions are important from both the standpoint of scoring and from that of analysis. The answer will be sought by using the scores in the Binet-Simon Tests[5] as a criterion and then using partial correlations.

[4] The lowest standard deviation is 15.56 and the highest 16.68. The exact standard deviation for each factor and test will be given later.

[5] Whenever the expression, "Binet-Simon Tests" is used, the Stanford Revision is meant.

Some objections could be raised to this method of procedure. The first one is the criterion itself. Does it include all factors in tests that have value? Does it include these in such a proportion that the scores are the best ones? No attempt will be made to answer these questions at this stage. From the point of view of this study, these are not such important questions, since the criterion as well as the experiments will be analyzed. In other words these are questions that should be answered after the experiments have been analyzed. The scores resulting from the combination of factors will have no value other than for analysis as far as this investigation is concerned. The tests have some other serious defects. Only those will be mentioned that apply to adults, since this investigation is limited entirely to this field. It is claimed that the Binet-Simon tests fail to distribute individuals in the upper quartile widely enough. In other words, the tests are too easy for very bright adults. The highest intelligence quotient possible is 122. Some of the individuals were not measured accurately. This is a serious objection to the tests themselves. For this study, however, it is not so serious as it would be where the object was to determine the actual intelligence quotients. It is true that the intelligence quotients have been used, but only the ranking of them was used in determining the percentiles. It is reasonable to expect that the ranking of the subjects by intelligence quotients is much more nearly correct than the absolute intelligence quotients themselves. Another objection is that the tests are not standardized for the upper years. This objection does not interfere in the present cases, since it is only the ranking of the subjects in intelligence quotients that is used, and since only one of the subjects made the highest score possible.

Other criteria could have been used. The school grades in the various subjects could have been combined and used. This was done by Rosenow.[6] That scheme, however, would not have worked very well in this case, since the subjects had different teachers during the year and there was no standard to unify the

[6] Rosenow, Curt, The Analysis of Mental Functions, *Psychol. Monog.*, 1917, *24* (No. 106).

grading. There are other reasons why academic marks are not good for a criterion. Too many other factors, such as health, outside attractions, interest in the subject, etc., which are very often not known by the teacher, enter into the grades of the students. The school grade is certainly not as significant as the Binet-Simon tests in determining intelligence. This fact has already been pointed out by psychologists.[7] Another criterion that is regarded as sound by some is the combined judgment of instructors.[8] This, however, has its limitations, especially for the present study, where no instructors could be selected who were acquainted with all the subjects. This, as has been shown by Thorndike,[9] is not an unsurmountable difficulty. Taking everything into consideration, however, there probably is no one thing that would make a better criterion than the Binet-Simon tests. This criterion could have been made much better if it had been supplemented with other tests and the combined results used. The Otis tests could have been given and the intelligence quotients combined with those in our criterion. This, however, was not thought of until it was too late to get the tests and administer them.

The second objection to this procedure is that it assumes linearity. It is claimed by some that correlations are meaningless when non-linearity exists. Rosenow[10] has given the best rebuttal to this argument that has come to the writer's notice. He says: "It follows that, taken merely as an indication that an actual relation does exist between two variables, r, the coefficient of correlation, is actually entitled to increased confidence if non-linear regression is shown. Indeed the mere proof of non-linear regression is in and of itself proof of the existence of a true relation, and also of the fact that it is greater than in-

[7] *E. g.*, Peterson, Joseph, The Rational Learning Test Applied to Eighty-one College Students, *J. of Educ. Psychol.*, 1920, *11*, 137 ff.

[8] Ruml, Beardsley, The Reliability of Mental Tests in the Division of An Academic Group, *Psychol. Monog.*, 1917, *24* (No. 105).

[9] Thorndike, E. L., Combining Incomplete Judgments of Relative Position, *J. Phil. Psychol. and Sc. Methods*, 1916, *13*, 197 ff.

[10] Rosenow, Curt, The Analysis of Mental Functions, *Psychol. Monog.*, 1917, *24*, (No. 106), 10 f.

dicated by r. It can hardly be claimed that a positive assertion which errs only on the conservative is meaningless. As a special case we may note that r=o does not necessarily indicate the absence of relation." Again he says: "The subject of non-linear regression for the psychologist amounts simply to this. If he is investigating the relation of two variables to each other he can get nearest the truth by 'fitting' a curve and determining its equation. Even in that case useful results are practically always obtainable by assuming linearity. But if one is dealing with a complex situation the only practical possibility with our present technique is to assume linearity. The results, when properly interpreted, will *not* be meaningless."

In this investigation linearity will be assumed for the purpose of calculating correlations and combining factors, but interpretations will be made in such a way as to make allowance for all cases of non-linearity. That is, the degree of non-linearity, as shown by an inspection of the curves through the means of the rows and columns, will be noted in each case.

After the factors in the tests have been analyzed in the light of the criterion, the investigation will be carried further by the use of multiple correlation. This may be done directly from the equation,[11]

$$R^2_{1(23456)} = 1-(1-r^2_{12})\,(1-r^2_{13.2})\,(1-r^2_{14.23})\,(1-r^2_{15.234})\,(1-r^2_{16.2345}) \qquad (1)$$

in which R is the symbol for the correlation between the criterion I and the factors 2, 3, 4, 5, and 6 combined in such a way as to give the highest correlation possible. It is possible by the use of this formula to determine what factors are necessary to get the highest correlation. If some factors are shown by partial correlations to have nothing in common with the criterion except what is contained in other factors and then the formula for multiple correlation shows that the correlation with the criterion is not reduced by discarding these factors, there is no reason for using them.

[11] Yule, G. U., *Introduction to The Theory of Statistics*, 248 ff.

Now that it has been decided just what factors should be used, the next step is to determine the best combination of them. Must they be combined in equal proportion or in some other ratio? To determine this question, the following formula will be used:

$$C = \frac{\sigma_M (r_{IM} r_{Mm} - r\ )}{\sigma_m (r_{Im} r_{Mm} - r_{IM})} \qquad (2)$$

in which M is the major factor, m the minor factor, I the criterion and C a constant which determines how m shall be weighted in order to give the highest correlation with I when combined with M. In order to find this highest correlation after C has been determined, the following formula will be used:

$$r_{I(M+Cm)} = \frac{r_{IM}\sigma M + Cr_{Im}\sigma m}{\sqrt{\sigma M^2 + 2Cr_{Mm}\sigma M\sigma m + C^2\sigma m^2}} \qquad (3)$$

in which the letters have the same meaning as in formula[12] (2). If more than two factors need to be combined in any test, two will be combined in the best way and then this result with the third factor. In this way any number of factors or tests may be combined.

The method of scoring will then consist of four steps: first, the determination of the significant factors by multiple and partial correlation; second, the finding of the best combination of the significant factors; third, the determination of the best correlation when this combination is used; fourth, actually combining the factors and working out the correlation to test the reliability of the mathematical work.

Each factor will be further analyzed by stating the theoretical relation to the criterion and the actual relation in terms of the curves of the means of the columns and of the rows. The tables of partial correlations will be freely used in determining the relation of each factor to the criterion and to the other factors. For example, an attempt will be made to find the relation of

[12] For the development of these formulae, see Thurstone, L. L., A Scoring Method for Mental Tests, *Psychol. Bull.*, 1919, *16*, 235 ff.

the time factor to the criterion and then to the other factors of repetitions and errors. Special cases that fall below the fortieth percentile in one test and above the sixtieth in another will be analyzed as far as objective data will permit.

III. Subjects

Eighty college students were tested in securing the data for this investigation. All were in the same course, conducted by the writer in the winter and spring terms of the school year 1919-20. The testing began early in January and continued until the first week in June. The course was conducted in three sections. One section was begun the first week in January and finished on the twenty-fifth of March. The other two were begun the last week in March and finished on the ninth of June. Six of the subjects tested are not included in the final list of seventy-four because of having failed to take all the tests or because of having been coached in one of the tests. A record of the sex and college class is given in Table I for each subject.

In many respects this represents a selected group. There is considerable uniformity in age, education, and social status. The majority are high school graduates, and those who are not have in some other kind of school completed work sufficient for college entrance. There are three or four in the group who have very low intelligence. These have browsed around here and there, gaining a few credits at each place, until they were able to get freshman standing in a college, but are not able to do real college work. No attempt was made to select subjects for the investigation. The aim was to use all who entered a certain class in education. This was followed as nearly as possible, the only exception being in the case of six students, of whom four left school before all the tests were taken and two showed evidence of having been coached on one of the experiments.

IV. The Binet-Simon Tests (*Stanford Revision*)

In every instance the subject was first given the tests for the average adult. If he passed on all of them he was then given the tests for the superior adult; but if he failed on one or more of

TABLE I. Showing Sex and College Class of Each Subject.

Subject[13]	Sex	College Class	Subject	Sex	College Class
1	f	Freshman	38	f	Freshman
2	f	"	39	m	"
3	m	"	40	f	Sophomore
4	f	"	41	f	Senior
5	m	Sophomore	42	f	Sophomore
5	f	Freshman	43	f	Freshman
7	f	"	44	f	"
8	m	"	45	m	"
9	f	"	46	m	Sophomore
10	m	"	47	f	Freshman
11	f	"	48	f	"
12	m	"	49	f	Sophomore
13	f	"	50	f	Freshman
14	m	"	51	m	"
15	f	"	52	f	"
16	f	"	53	f	"
17	m	"	54	f	"
18	f	"	55	f	"
19	f	Senior	56	m	"
20	f	Freshman	57	f	"
21	f	"	58	f	"
22	f	Sophomore	59	f	Sophomore
23	f	Freshman	60	f	Freshman
24	f	"	61	f	"
25	f	"	62	f	"
26	f	"	63	f	"
27	f	"	64	f	"
28	f	Sophomore	65	f	"
29	f	Junior	66	m	Freshman
30	f	Freshman	67	f	Sophomore
31	f	Sophomore	68	m	"
32	f	"	69	f	"
33	m	Freshman	70	f	Freshman
34	m	"	71	f	"
35	f	Sophomore	72	f	"
36	f	Freshman	73	f	"
37	f	"	74	f	"

TABLE II. Showing Summary of Subjects.

	Freshman	Sophomore	Junior	Senior	Totals
Male	13	3	0	0	16
Female	44	11	1	2	58
Totals	57	14	1	2	74

[13] The numbers assigned to the several subjects in this table will be used in all following tables.

them he was then given the tests for fourteen years of age. In case all the tests in this year were not passed, the twelve year tests were given. In no case was it necessary to go further back and in very few back so far. If the subject did one or more of the tests for the average adult correctly, he was given the tests for the superior adult also. The mental age and the intelligence quotient were found for each subject in the manner suggested by Terman.[14] Table III gives the intelligence quotient and the percentile rank for each subject.

TABLE III. Showing Intelligence Quotient and Percentile Rank for Each Subject in the Binet-Simon Tests.

Subject	Intelligence Quotient	Percentile Rank	Subject	Intelligence Quotient	Percentile Rank
1	113	69	38	82	21
2	116	78	39	110	63
3	98	42	40	96	39
4	106	54	41	107	58
5	122	86	42	109	60
6	91	30	43	94	35
7	104	52	44	95	37
8	76	0	45	99	45
9	80	18	46	110	63
10	91	30	47	102	49
11	113	69	48	101	47
12	99	45	49	110	63
13	95	37	50	103	51
14	96	39	51	98	42
15	106	54	52	107	58
16	106	54	53	93	34
17	83	23	54	101	47
18	105	53	55	101	47
19	113	69	56	98	42
20	96	39	57	104	52
21	92	32	58	110	63
22	95	37	59	101	47
23	98	42	60	78	14
24	116	78	61	116	78
25	98	42	62	113	69
26	110	63	63	113	69
27	99	45	64	103	51
28	101	47	65	107	58
29	104	52	66	113	69
30	102	49	67	107	58
31	107	58	68	113	69
32	93	34	69	107	58
33	87	27	70	116	78
34	113	69	71	85	25
35	99	45	72	110	63
36	93	34	73	104	52
37	109	60	74	91	30

[14] Terman, *The Measurement of Intelligence*, 1916.

V. The Rational Learning Test

(1) Description of Test and Method of Scoring.

In this test the procedure of the author[15] was followed as nearly as possible. The instructions to the subject and the method of recording the data were identical. Some variations occur in the scoring, as will appear later. The instructions to the subject follow:

"This is a memory-reason test. The letters A, B, C, D, E, F, G, H, I and J are numbered in a random order from 1 to 10. I call out the letters in their order and you are to guess for each letter till you get the correct number, when I say 'Right.' Then I call out the next letter, and so on. This is continued until you get each number right the first guess twice in succession through the series, from A to J. Then you are through. You must ask no questions, but are to use all the mental powers at your command. You will be judged by (1) the total time you take, (2) the number of errors or wrong guesses you make (every number you speak being a guess), and (3) the number of repetitions from A to J that you require for the learning. Re-read these instructions carefully, if necessary, to understand what you are to do. The meaning will be clearer as we go on with the experiment."

The subject was seated at a table opposite the experimenter and shielded from the latter by a screen. He was given a typewritten copy of the instructions and allowed to read and study them until he was ready to go on with the learning. He usually consumed about one minute. When the subject said he was ready to proceed, a stop watch was started, the letters called out in their order, and the responses recorded as shown in Table IV. When the learning was complete, the total time was recorded and the subject asked to write as much as he could about how he learned to repeat the numbers in order.

[15] Peterson, Joseph, Experiments in Rational Learning, *Psychol. Rev.*, 1918, *25*, 433 ff.

TABLE IV. Showing Record of Subject Number 68 and Method of Recording Data.

Letters	A	B	C	D	E	F	G	H	I	J	Errors			
Numbers	6	4	9	1	8	10	3	2	7	5	Uc.	†	*	Total
First Repetition	6	3 5 9 8 7 4	1 3 2 5 6† 7 8 9	10 8 7 5 3 2 1	2 3 5 7 8	2 3 5 7 8† 10	2 3	2	9† 7	5	29	3	0	32
Second Repetition	6	4	9	2 1	8	3 4† 5 2 7 8† 9† 5* 10	3	8† 7 3† 2	3† 5 7	3† 5	15	7	1	23
Third Repetition	6	4	9	1	10 8	9† 2 3 7 8† *9† 10	7 8† 2 1† 3	7 9† 2	7	5	13	6	1	20
Fourth Repetition	6	4	9	1	10 8	2 7 8† 9† 10	3	2	7	5	5	2	0	7
Fifth Repetition	6	4	8 9	1	10 8	2	3	2	7	5	2	0	0	1
Sixth Repetition	6	4	9	1	10 8	10	3	2	7	5	1	0	0	1
Seventh Repetition	6	4	9	1	8	10	3	2	7	5	0	0	0	0
Eighth Repetition	6	4	9	1	8	10	3	2	7	5	0	0	0	0
Totals											65	18	2	85

Three schedules of numbers were used in giving this test. Subjects 12, 15, 20, 21, 22, 25, 31, 34, 35, 37, 39, 40, 44, 47, 49, 52, 53, 54, 57, 59, 62, 66, 67, 68, 69, and 74 used schedule 1.[16] Subjects 1, 2, 5, 6, 7, 8, 10, 13, 14, 17, 18, 19, 28, 29, 30, 36, 38, 50, 55, 56, 60, 63, 70, 72, and 73 used schedule 2.[17] Subjects 3, 4, 9, 11, 16, 23, 24, 26, 27, 32, 33, 41, 42, 43, 45, 46, 48, 51, 58, 61, 64, 65 and 71 used schedule 3.[18]

The numbers were arranged according to no special method. Schedule 2 is that used by Dr. Peterson[19] in some of his work. Schedules 1 and 3 were made with the idea of having them equal in difficulty to 2. This, of course, makes the statement as to random numbering in the directions to the subject slightly incorrect, but each schedule could be obtained by a random selection. Therefore the statement should put the subject to no disadvantage. From observation there is no difference in the difficulty of the schedules. The orders 3-2, 5-4, and 6-5 in schedules 1, 2, and 3 respectively present especial difficulty to many subjects. The purpose in having different schedules was to reduce probability of coaching. There was no evidence of coaching in this test. Each subject was asked after the test was performed not to tell any other member of the class anything that might assist in the learning.

The column headed Uc. in this table gives the unclassified errors, or the total number of errors regardless of kind.

Errors marked † are called logical errors. They are errors which consist in guessing a number that has already been used for an earlier letter of the series, one that could, therefore, not possibly be right.

Errors marked * are called perseverative errors. They are errors which consist in repeating a wrong guess while reacting to a single letter.

[16] Schedule 1	A	B	C	D	E	F	G	H	I	J
	6	4	9	1	8	10	3	2	7	5
[17] Schedule 2	A	B	C	D	E	F	G	H	I	J
	9	6	2	10	8	1	5	4	7	3
[18] Schedule 3	A	B	C	D	E	F	G	H	I	J
	4	8	3	1	9	7	10	6	5	2

[19] Peterson, Joseph, Experiments in Rational Learning, *Psychol. Rev.*, 1918, *25*, 433 ff.

In Table V are given the raw scores of all the subjects, and also, on the right hand side of the table, the scores converted into percentile rank and the final combined score. This combined score for the test is determined by combining the percentile rank in repetitions and in perseverative errors and then converting into percentile score from Rugg's table, as will presently be explained. Tables VI and VII show, respectively, the total and the partial correlations of each factor in the Rational Learning Test with the criterion, the Binet-Simon tests, and the total and the partial correlations of the factors in the Rational Learning Test.

TABLE V. Showing the Number of Minutes†, the Number of Repetitions, the Number of Each Kind of Errors, and the Percentile Rank for Each Kind of Data in Rational Learning.

	Raw Score in					Percentile Rank in					
Subject	Time	Rep.	Uc.E.	L.E.	P.E.	Time	Rep.	Uc.E.	L.E.	P.E.	Score‡
1	3	4	27	0	0	86	76	72	82	71	78
2	17	8	124	64	3	42	46	35	30	55	52
3	14	7	96	34	13	46	53	42	45	35	40
4	18	8	71	8	3	40	46	50	63	55	52
5	5	4	56	23	0	74	76	57	49	71	78
6	6	3	23	0	0	72	85	76	82	71	84
7	17	11	64	13	1	42	35	53	57	63	49
8	21	14	196	78	38	31	25	21	21	23	18
9	12	7	21	2	0	53	53	79	73	71	65
10	19	11	113	34	15	36	35	37	45	28	28
11	18	9	47	10	3	40	42	61	59	55	48
12	20	14	240	143	12	33	25	18	18	38	28
13	7	5	59	13	4	69	68	55	57	51	61
14	20	11	168	66	3	33	35	28	28	65	44
15	16	6	53	28	3	44	61	59	47	55	59
16	7	4	44	6	4	69	76	65	66	51	68
17	25	15	169	72	11	25	16	27	23	40	22
18	14	9	81	28	5	46	42	46	47	48	44
19	9	7	93	38	7	61	53	43	42	45	49
20	13	10	62	18	5	49	38	54	53	48	38
21	12	9	79	42	3	53	42	46	38	55	48
22	8	6	105	44	6	65	61	40	35	46	56
23	18	12	130	58	0	40	31	31	32	71	53
24	8	8	43	9	2	65	46	65	61	59	56
25	17	14	109	31	3	42	25	39	46	55	36
26	16	10	88	34	17	24	38	45	45	25	28
27	12	7	88	35	4	53	53	45	43	51	54
28	12	7	53	16	0	53	53	59	54	71	65
29	14	7	75	21	6	46	53	48	50	46	51
30	10	6	35	1	4	59	61	69	76	51	58
31	33	14	110	36	13	0	25	38	43	35	25
32	12	7	72	40	2	53	53	49	40	59	59

TABLE V (Continued)

	Raw Score in					Percentile Rank in					
Subject	Time	Rep.	Uc.E.	L.E.	P.E.	Time	Rep.	Uc.E.	L.E.	P.E.	Score‡
33	18	12	116	38	11	40	31	36	42	40	34
34	13	13	129	41	2	49	28	32	38	59	40
35	10	5	45	5	2	59	68	63	68	58	68
36	19	7	87	36	13	36	53	45	43	35	40
37	8	8	47	9	0	65	46	61	61	71	60
38	30	11	178	71	13	18	35	23	25	35	33
39	5	3	23	4	0	74	85	76	71	71	84
40	11	6	75	23	10	57	61	48	49	40	52
41	9	5	41	7	0	61	68	67	65	71	73
42	20	12	174	54	13	33	31	25	33	35	32
43	11	6	38	10	1	57	61	68	59	63	65
44	17	8	106	20	10	42	46	40	51	40	38
45	19	6	65	19	12	36	61	52	52	38	51
46	13	5	48	13	3	49	68	60	57	55	62
47	22	9	126	47	28	29	42	33	35	23	31
48	12	9	62	7	5	53	42	54	65	48	44
49	9	5	44	14	2	61	68	65	56	59	68
50	19	10	146	43	35	36	38	30	36	18	22
51	24	9	111	42	14	27	42	38	38	31	35
52	12	7	55	10	4	53	53	58	59	51	54
53	11	9	90	40	8	57	42	43	40	44	38
54	13	8	56	12	5	49	46	57	58	48	45
55	19	9	98	59	5	36	42	41	31	48	44
56	32	15	463	267	73	14	16	0	0	0	0
57	26	21	398	206	28	23	0	14	14	23	14
58	10	7	59	18	9	59	53	55	53	42	46
59	15	8	97	48	9	45	46	42	34	42	40
60	7	8	24	5	0	69	46	73	68	71	60
61	14	5	72	23	9	46	68	49	49	42	57
62	8	6	45	7	1	65	61	63	65	63	65
63	4	6	29	5	0	82	61	70	68	71	70
64	8	7	41	4	0	65	53	67	71	71	65
65	5	5	17	0	0	74	68	86	82	71	73
66	11	6	69	19	2	57	61	51	52	59	61
67	14	11	124	68	9	46	35	35	27	42	35
68	8	8	65	18	2	65	46	52	53	59	56
69	12	7	70	14	8	53	53	51	56	44	48
70	5	5	20	1	0	74	68	82	76	71	73
71	9	5	58	10	13	61	68	56	59	35	53
72	28	6	76	20	15	21	61	47	51	28	42
73	22	7	116	24	14	29	53	36	48	31	37
74	13	7	54	8	8	49	53	58	63	44	48

† The time is given to the nearest minute.

‡ Scores are found by adding the percentile rank in repetitions to the percentile rank in perseverative errors and then again reducing to absolute percentiles by Rugg's table. The reason for this will appear later.

TABLE VI. Showing Correlations† and Partial Correlations of Each Factor‡ of the Rational Learning Test with the Binet-Simon Tests. The symbol for correlation has been omitted. The table should read $r_{12} = .27$, $r_{12.3} = .08$, etc.

12	.27	13	.31	14	.25	15	.23	16	.27
12·3	.08	13·2	.14	14·2	.05	15·2	.04	16·2	.15
12·4	.14	13·4	.17	14·3	.02	15·3	.02	16·3	.18
12·5	.16	13·5	.20	14·5	.11	15·4	—.02	16·4	.17
12·6	.10	13·6	.18	14·6	.03	15·6	.05	16·5	.19
12·34	.09	13·24	.14	14·23	—.02	15·23	—.02	16·23	.16
12·35	.08	13·25	.14	14·25	.04	14·24	—.01	16·24	.14
12·36	—.02	13·26	.15	14·26	—.03	15·26	.01	16·25	.14
12·45	.14	13·45	.17	14·35	.00	15·34	.00	16·34	.22
12·46	.10	13·46	.22	14·36	—.14	15·36	—.07	16·35	.18
12·56	.09	13·56	.19	14·56	—.02	15·46	.04	16·45	.17
12·345	.09	13·245	.14	14·235	—.01	15·234	.00	16·234	.20
12·346	.01	13·246	.20	14·236	—.14	15·236	—.07	16·235	.16
12·356	.00	13·256	.16	14·256	—.03	15·246	.04	16·245	.14
12·456	.10	13·456	.24	14·356	—.15	15·346	.08	16·345	.23
12·3456	.01	13·2456	.21	14·2356	—.14	15·2346	.08	16·2345	.21

† All correlations are worked by the product-moment method.

‡ For the sake of brevity the factors are designated by numbers as follows:

1. Binet-Simon Tests
2. Time
3. Repetitions
4. Unclassified Errors
5. Logical Errors
6. Perseverative Errors

TABLE VII. Showing Correlations and Partial Correlations of the Factors in the Rational Learning Test.

23	.72	24	.78	25	.72	26	.69	34	.78
23·4	.28	24·3	.50	25·3	.43	26·3	.55	34·2	.50
23·5	.43	24·5	.46	25·4	—.07	26·4	.26	34·5	.48
23·6	.60	24·6	.55	25·6	.26	26·5	.43	34·6	.72
23·45	.27	24·35	.30	25·34	—.04	26·34	.34	34·25	.34
23·46	.37	24·36	.22	25·36	.24	26·35	.44	34·26	.57
23·56	.44	24·56	.28	25·46	.00	26·45	.23	34·56	.55
23·456	.37	24·356	.03	25·346	.10	26·345	.34	34·256	.50
35	.71	36	.49	45	.94	46	.75	56	.64
35·2	.40	36·2	.00	45·2	.87	46·2	.47	56·2	.29
35·4	—.12	36·4	—.23	45·3	.88	46·3	.67	56·3	.45
35·6	.59	36·5	.07	45·6	.91	46·5	.59	56·4	—.30
35·24	—.10	36·24	—.32	45·23	.84	46·23	.55	56·23	.30
35·26	.41	36·25	—.14	45·26	.88	46·25	.47	56·24	—.28
35·46	—.24	36·45	—.30	45·36	.87	46·35	.64	56·34	—.33
35·246	—.22	36·245	—.36	45·236	.85	46·235	.58	56·234	—.34

If we consider the size of the probable error,[20] it seems safe to infer from the data in Table VI that repetitions and persever-

[20] The probable error for 74 cases is .075 for zero correlation and .063 for a correlation of .40. The probable errors are not given in each instance, since they are available in tables.

ative errors have elements in common with the criterion that are not common to the other factors or to each other. This conclusion is based on the correlations, $r_{13\cdot2456}$ and $r_{16\cdot2345}$, which are .21 in each case. This is three times the probable error and may be regarded as significant. Then if it is desired to score the Rational Learning Test so as to have it correlate highest with the Binet-Simon tests, these two factors must be included. It is evident that time, unclassified errors, and logical errors, have nothing in common with the criterion that is not included in the other four factors, if we hold that a correlation less than three times the probable error is not significant. It must also be kept in mind in this case that linearity is assumed. To be more specific, the expression, $r_{12\cdot3456}=.01$, means that everything common to 1 and 2 is contained in 3, 4, 5 and 6. Then factor 2, time, may be discarded if the remaining four are used. Further, since $r_{15\cdot346}=.08$, factor 5, logical errors, may be discarded. In like manner, since $r_{14\cdot36}=-.14$, it is fairly safe to discard factor 4, unclassified errors. It is true that a correlation of -.14 may be slightly significant. It may mean that, if the elements in repetitions and perseverative errors are removed, the more intelligent the subject the more errors he will make. Let the question be pushed further by the use of multiple correlation. By the use of formula (1), it is found that

$$R_{1(23456)} = .33$$
$$\text{But } R_{1(36)} = .32$$

The difference between the correlation with the criterion when all five factors are used and when repetitions and perseverative errors only are used is small enough to be entirely neglected. This is further evidence that factors 3 and 6 are sufficient to use in the final scores. The correlation would be lowered, if anything were lost by discarding the others.

The next step is to find the proper combination of these two factors to give the best correlation. That is, must they be combined equally or in some other proportion? To determine this, formula (2) will be used. Repetitions will be designated as the

major factor and perseverative errors as the minor factor. The fundamental constants for determining C are:

$$r_{IM} = .31$$
$$r_{Im} = .27$$
$$r_{Mm} = .49$$
$$\sigma_M = 16.35$$
$$\sigma_m = 15.56$$

When these values are substituted in formula (2), the result is 1.05, or for all practical purposes 1. This means that the two combined equally will give the best correlation. To determine what the correlation will be when the two factors are combined equally, formula (3) will be used, in which the letters have the same meaning as in formula (2). The correlation is found to be .33. The actual correlation when the scores in repetitions and perseverative errors are combined equally is .324.

The final scores are found by adding the percentile rank in repetitions to the percentile rank in perseverative errors and then again using Rugg's table for reducing to absolute percentiles as was done with the raw scores.

Multiple correlation indicated that a correlation of .32 could be obtained by combining repetitions and perseverative errors. Formula (2) indicated that the best combination is that in which they are combined equally. Formula (3) indicated that a correlation of .33 should be obtained when they are combined equally. When the actual combination is made and the correlation is worked out, the result is .324. The difference between these two numbers is small enough to be accounted for by the way fractions are carried out, and by using 1 instead of 1.05 as a ratio for combining.

(2) Analysis of Data.

There is a "present but low"[21] positive correlation in this test

[21] Correlations below .20 will be considered negligible; from .20 to .40, present but low; from .40 to .60, marked; above .60, high. See Rugg, H. O., *Statistical Methods Applied to Education*, 1917, 256.

between the criterion and each of the factors. In other words, there is slight evidence that ability in the criterion and ability in each factor of the test accompany each other. Subjects above the average in the former will be above the average in each of the latter. Since the variability[22] is made the same in each kind of data by reducing to absolute percentiles, a change in one test or factor will be accompanied by a like change in the other test or factor, and equal to the correlation of the two tests or factors. Thus, time and the criterion have a correlation of .27. Therefore, every unit-change in one will be accompanied by a like change of .27 of a unit in the other. In like manner, each of the other factors may be compared with the criterion by reference to the correlations in Table VI. When the standard deviations are equal, the regression line for the columns takes the form $y = rx$ and that for the rows takes the form $x = ry$.

Each factor in this test has something common to the criterion. Repetitions have most and unclassified errors least. There is, moreover, a great overlapping of common elements. Thus repetitions contain all that is common to time and the criterion, all that is common to unclassified errors and the criterion, and all that is common to logical errors and the criterion. Time has all that is in logical or unclassified errors with respect to the criterion. Unclassified and logical errors have practically the same elements as far as the criterion is concerned. Repetitions and perseverative errors have something, however, not found in any one of the other factors or in any combination of them. These two contain everything in all the factors needed to get the highest correlation with the criterion.

We shall use Blakeman's criterion[23] for linearity. When the proper values substituted in the formula give a result greater than 2.5, non-linearity will be said to exist and a table will be constructed showing the lines of the means of the rows and the

[22] The standard deviations for the Binet-Simon Tests, time, repetitions, unclassified errors, logical errors and perseverative errors are respectively 16.62, 16.47, 16.36, 16.68, 16.56, and 15.56.

[23] For formula, see Rugg, *Statistical Methods Applied to Education*, 1917, 283.

means of the columns. If the result is less than 2.5, the correlation is for all practical purposes linear and no table of regression lines will be constructed.

The correlation-ratios for each of the factors with the criterion are as follows:

12 = .46 and .52
13 = .41 and .47
14 = .47 and .52
15 = .44 and .49
16 = .37 and .40

These values substituted in the Blakeman formula give results as follows:

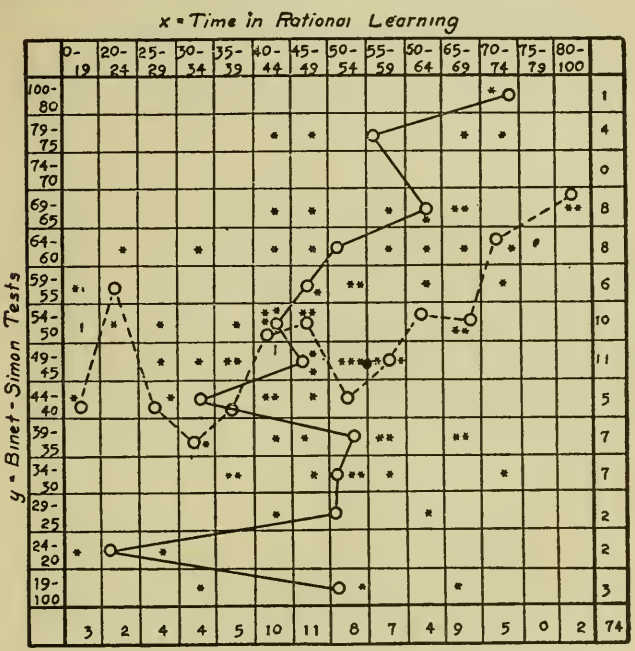
Tests 1 and 2, 2.37 and 2.83
Tests 1 and 3, 1.71 and 2.25
Tests 1 and 4, 2.54 and 2.90
Tests 1 and 5, 2.39 and 2. 76
Tests 1 and 6, 1.61 and 1.88

According to Blakeman's criterion, the correlations of repetitions and perseverative errors with The Binet-Simon tests are linear, but the correlations of time, unclassified, and logical errors with The Binet-Simon tests are non-linear. The regression lines for the last three correlations are now ready to be constructed.

The regression lines showing the correspondence between scores in time and the criterion are shown in Table VIII. Each asterisk indicates the position of one person as determined by both tests. The circles through which the broken line passes represent the means[24] of the columns and those through which the whole line passes represent the means of the rows. This table shows that the two sets of relationships are in very close agreement. That is, the regression of the x-values on y is very nearly the same as the y-values on x. The x-values show a tendency to increase constantly with an increase in y-values from the fortieth percentile up. Below this point there is little relation between x and y-values. The y-values have a tendency to increase constantly

[24] The means for the columns or rows are found by adding the exact percentile ranks in each row or column and dividing by the number of cases.

TABLE VIII. Showing the Distribution of Subjects on the Basis of Time in Rational Learning and the Binet-Simon Tests. Circles through which the broken line passes represent the means of the columns and those through which the continuous line passes represent the means of the rows. Each asterisk represents a subject.

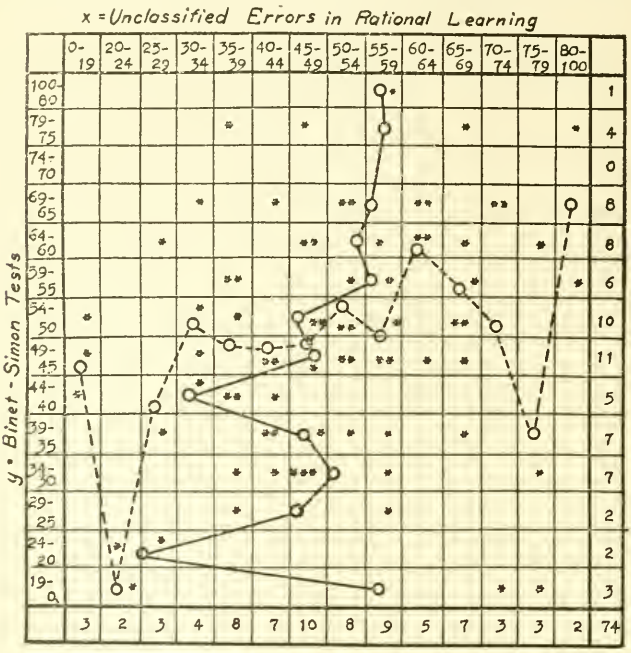


with an increase in x-values from the fiftieth percentile up. Below this point there is little relation between y and x-values.[25]

The relation between unclassified errors and the criterion is shown in Table IX. The agreement between the regression of the x-values on y and the y-values on x is not close. In neither case is the tendency for one value to increase with an increase in the other constant. The x-values increase with an increase in y-values from the fortieth percentile up to the sixtieth. The other regression line is almost horizontal from the thirtieth to the sixtieth percentile. An increase in Binet-Simon scores indicates nothing with respect to unclassified errors until the fortieth percentile is reached. An increase in Binet-Simon scores

[25] Reasons for this lack of relation in the lower quartile will be suggested later in comparing the final scores with the criterion.

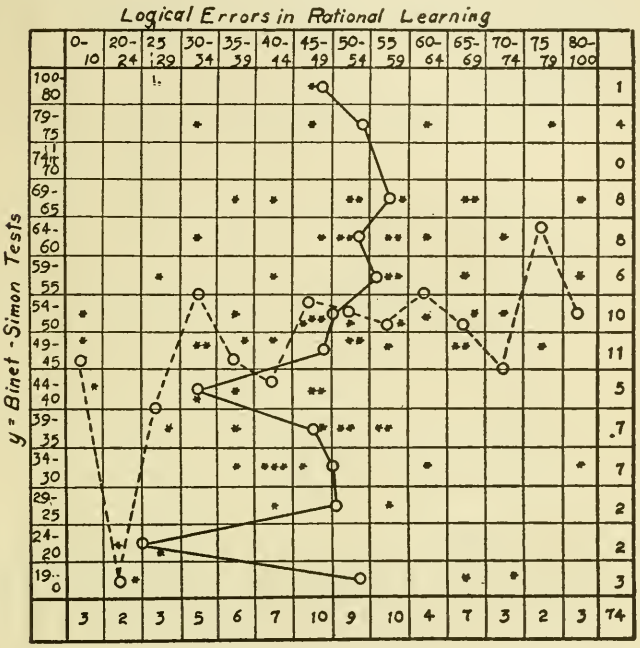
TABLE IX. Showing the Distribution of Subjects on the Basis of Unclassified Errors in Rational Learning and the Binet-Simon Tests. Circles through which the broken line passes represent the means of the columns and those through which the continuous line passes represent the means of the rows. Each asterisk represents a subject.



from the fortieth to the sixtieth percentile means a rather rapid increase in unclassified errors. Above the sixtieth percentile an increase in Binet-Simon scores means no change in unclassified errors. From the beginning to the end, an increase in unclassified errors means nothing with respect to intelligence as measured by the Binet-Simon Tests.

Table X shows the relation existing between logical errors and the criterion. The agreement between the two sets of values is not close. There is a fairly constant increase in y-values with an increase in x-values from the fortieth percentile up. A smoothed curve will show an increase from the very beginning. In the other regression line there is no tendency for the x-values to increase constantly. Up to the fortieth percentile an increase in Binet-Simon scores means no change in score in

TABLE X. Showing the Distribution of Subjects on the Basis of Logical Errors in Rational Learning and the Binet-Simon Tests. Circles through which the broken line passes represent the means of the columns and those through which the continuous line passes represent the means of the rows. Each asterisk represents a subject.



logical errors, but from here on to the sixty-fifth percentile, it means a constant increase. Above this point there are only five cases, and the curve has no significance.

The skewness of each distribution is almost zero, since the median and the mean almost coincide when the scores are reduced to percentiles. This assumes that skewness is measured in terms of the median, the mean, and the standard deviation. All medians and means are approximately 50, and all standard deviations are approximately 16.66.

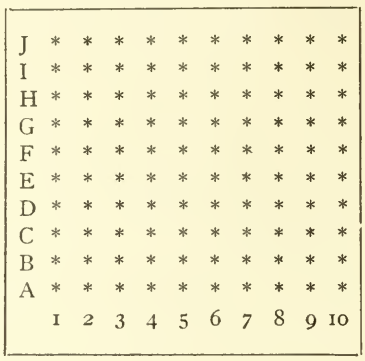
VI. THE RATIONAL LEARNING TEST (*Modified*)

(1) Description of Test and Method of Scoring.

This test is very similar to Rational Learning. The apparatus consists of a board about twenty inches square, through which

are put one hundred bolts arranged in ten rows with ten bolts in a row. The rows are lettered from A to J and the bolts in each are numbered 1 to 10. One bolt, and only one, in each row is connected in a circuit with an electric bell so that when this bolt is touched with a stylus the bell will ring. Figure I shows the apparatus.

Fig. I. Showing Apparatus for Rational Learning (Modified).



Each asterisk represents a bolt. The bolts are actually numbered in each row just as indicated in row A in the diagram. The method of recording the data is exactly as in Rational Learning. The instructions to the subject follow:

"You have in the apparatus before you ten rows of bolts with ten bolts in each row. The rows are lettered from A to J and the bolts in each row are numbered from 1 to 10. One bolt in each row, and only one, is connected in such a way that the bell will ring when the circuit is made. That is, each letter is assigned a number in a random order from 1 to 10. This number is the one that will cause the bell to ring when the bolt is touched. No two letters have the same number.

Your problem is to begin with row A and find the bolt that will ring the bell. Then go on to row B and find the bolt. Continue until you have reached row J. Now go back to row A and repeat the process. Continue repeating the process until you go from A to J twice in succession without making any mistake. Then you are through.

You are to ask no questions after you start, but are to use all the mental powers at your command in order to complete the learning as soon as possible. You will be judged by (1) the total time you take, (2) the number of errors or wrong guesses you make (every bolt you touch being a guess), (3) the number of repetitions from A to J that you require for the learning. Re-read these instructions carefully, if necessary, to understand what you are to do. The meaning will be clearer as we go on with the experiment."

The subject stood facing the apparatus so that row A was toward him. The experimenter recorded every response just as was done in Rational Learning. When the test was finished the total time was recorded and the subject was asked to write as much as he could about the method he used in learning the problem.

The same three schedules of numbers were used in this test as in Rational Learning.[26] Those having schedule 1 in Rational Learning had schedule 2 in this one; those having schedule 3, had 1; and those having schedule 2, had 3. The same precaution was taken to prevent coaching as in Rational Learning. Each subject was asked not to tell any other member of the class anything that might assist in the learning. Results are given in Table XI.

Table XII shows that time has something in common with the criterion that is not common to the other factors. Perseverative errors seem to be least significant and may be dropped as far as the final scores are concerned. If we consider only the four factors and partial correlations of the third order, logical errors appear useless. In like manner, when the three remaining factors are considered alone, repetitions appear insignificant. If we push the analysis further, however, it is very evident that unclassified errors must be retained with time to make up the final score. We may make the analysis more logical by first considering $r_{16.2345} = .01$, which means that there is nothing common to 1 and 6 that is not contained in 2, 3, 4, and 5. Factor 6 may therefore be dropped. In like manner, since $r_{15.234} = .03$

[26] This test was given before Rational Learning.

TABLE XI. Showing the Number of Minutes, the Number of Repetitions, the Number of each Kind of Errors, and the Percentile Rank for Each Kind of Data in Rational Learning (Modified).

	Raw Score in					Percentile Rank in					
Subject	Time	Rep.	Uc.E.	L.E.	P.E.	Time	Rep.	Uc.E.	L.E.	P.E.	Score‡
1	3	4	47	19	0	82	75	75	69	76	79
2	7	10	138	72	3	68	51	43	41	58	56
3	7	4	42	18	3	68	75	82	71	58	77
4	10	7	55	17	3	57	65	69	73	58	67
5	2	3	45	16	0	87	82	77	76	76	85
6	11	7	70	24	3	53	65	63	63	58	59
7	9	10	102	16	13	61	51	53	76	38	58
8	18	12	182	67	21	37	42	35	43	28	35
9	27	10	124	60	8	18	51	46	46	46	32
10	15	9	130	61	5	43	56	44	46	53	43
11	13	13	115	58	9	46	39	49	48	45	48
12	10	9	69	23	0	57	56	64	65	76	61
13	10	12	164	90	5	57	42	40	33	53	49
14	11	10	142	58	11	53	51	42	48	41	48
15	10	7	67	37	2	57	65	65	58	63	64
16	4	4	59	23	0	79	75	68	65	76	73
17	17	18	261	94	16	40	16	21	32	34	29
18	10	7	66	27	15	57	65	65	61	36	61
19	7	10	145	66	4	68	51	42	43	55	55
20	16	16	176	72	22	41	27	35	41	25	38
21	12	8	128	79	6	49	59	45	38	51	46
22	7	7	95	55	1	68	65	54	49	68	64
23	17	15	167	83	2	40	33	38	35	63	40
24	6	3	48	22	3	74	82	73	67	58	73
25	15	12	114	38	11	43	42	50	57	41	46
26	6	6	73	22	2	74	70	61	67	63	70
27	9	9	92	48	6	61	56	55	54	51	59
28	9	7	116	67	5	61	65	49	43	53	55
29	11	10	124	50	8	53	51	46	52	46	49
30	7	7	43	26	2	68	65	79	62	63	73
31	28	12	165	83	24	14	42	39	35	21	23
32	19	12	123	55	10	33	42	47	49	43	40
33	14	16	207	141	15	45	27	32	18	36	39
34	12	10	118	65	3	49	51	48	44	58	49
35	23	14	217	122	47	24	37	30	21	0	25
36	29	14	236	110	14	0	37	25	26	37	0
37	10	9	106	49	2	57	56	52	53	63	54
38	18	14	314	152	30	37	37	0	14	14	14
39	5	3	25	4	0	77	82	87	87	76	85
40	13	10	127	58	10	46	51	45	48	43	45
41	11	10	74	26	4	53	51	51	62	55	53
42	9	12	73	23	0	61	42	61	65	76	64
43	12	11	110	54	13	49	46	51	50	38	51
44	23	15	292	153	24	24	33	14	0	21	18
45	19	6	78	34	6	33	70	59	59	51	45
46	11	7	85	49	3	53	65	58	53	58	56
47	18	16	231	109	11	37	27	28	28	41	33
48	15	8	71	29	8	43	59	62	60	46	53
49	12	15	109	44	18	40	33	51	54	31	51

TABLE XI (Continued)

Subject	Raw Score in					Percentile Rank in					
	Time	Rep.	Uc.E.	L.E.	P.E.	Time	Rep.	Uc.E.	L.E.	P.E.	Score‡
50	7	8	95	52	6	68	59	54	52	51	64
51	24	12	250	110	17	21	42	23	26	32	21
52	21	17	189	71	24	28	21	33	42	21	29
53	12	10	154	87	12	49	51	40	34	39	44
54	12	8	48	8	2	49	59	73	79	63	64
55	19	11	235	95	21	33	46	27	30	28	27
56	17	14	172	78	7	40	37	36	38	49	38
57	9	8	131	62	0	61	59	43	45	76	53
58	13	18	269	117	16	46	16	18	23	34	32
59	19	15	151	83	2	33	33	41	35	63	36
60	15	16	207	74	19	43	27	32	40	30	37
61	13	13	59	5	16	46	39	68	82	34	58
62	9	8	100	43	7	61	59	54	55	49	58
63	19	15	166	81	11	33	33	38	37	41	34
64	19	12	91	29	7	33	42	56	60	49	44
65	11	11	122	76	1	53	46	48	39	68	51
66	19	9	105	41	2	33	56	52	56	63	42
67	14	16	167	64	9	45	27	38	45	45	42
68	7	11	88	37	6	68	46	58	58	51	67
69	12	11	104	53	11	49	46	53	51	41	52
70	7	10	61	20	2	68	51	66	68	63	69
71	17	24	188	94	9	40	0	34	32	45	36
72	22	7	91	41	4	27	65	56	56	55	42
73	8	10	88	52	1	64	51	58	52	68	64
74	8	6	52	18	0	64	70	70	71	76	69

† Scores are found by adding the percentile rank in time to the percentile rank in unclassified errors and then again reducing to absolute percentiles by Rugg's table. The reason for this will appear later.

TABLE XII. Showing Correlations and Partial Correlations of Each Factor† of Rational Learning (Modified) with the Binet-Simon Tests.

12	.42	13	.31	14	.44	15	.42	16	.35
12·3	.31	13·2	.08	14·2	.24	15·2	.23	16·2	.11
12·4	.19	13·4	—.05	14·3	.33	15·3	.30	16·3	.21
12·5	.23	13·5	.04	14·5	.15	15·4	.04	16·4	.07
12·6	.27	13·6	.13	14·6	.29	15·6	.28	16·5	.12
12·34	.20	13·24	.08	14·23	.24	15·23	.21	16·23	.08
12.35	.23	13·25	—.04	14·25	.08	15·24	.04	16·24	.00
12·36	.24	13·26	.04	14·26	.20	15·26	.20	16·25	.04
12·45	.19	13·45	—.05	14·35	.15	15·34	.04	16·34	.07
12·46	.17	13·46	—.06	14·36	.27	15·36	.25	16·35	.12
12·56	.19	13·56	.00	14·56	.10	15·46	.05	16·45	.07
12·345	.20	13·245	—.08	14·235	.11	15·234	.03	16·234	.01
12·346	.19	13·246	—.09	14·236	.23	15·236	.20	16·235	.04
12·356	.19	13·256	—.05	14·256	.07	15·246	.04	16·245	.01
12·456	.17	13·456	—.05	14·356	.11	15·346	.04	16·345	.08
12·3456	.19	13·2456	—.08	14.2356	.10	15·2346	.03	16·2345	.01

†For the sake of brevity the factors are designated by numbers as follows:

1. Binet-Simon Tests
2. Time
3. Repetitions
4. Unclassified Errors
5. Logical Errors
6. Perseverative Errors

factor 5 may be discarded. Since $r_{13.24} = .08$, factor 3 may be discarded. But since $r_{12.4} = .19$, and $r_{14.2} = .24$, it is evident that factors 2 and 4 must be retained.

This conclusion may be further verified by using multiple correlation. The following results are found:

$$R_{1(23456)} = .479$$
$$R_{1(24)} = .473$$

It is seen from the above that very little is lost by excluding all the factors except time and unclassified errors. We shall use for our final scores, then, these two factors which appear to give everything that is necessary to get the highest correlation with the criterion.

Our next problem is to find the combination of time and unclassified errors that will give this best correlation. The same formulae will be used as were used in Rational Learning. Designating unclassified errors by M and time by m, we may use the following data for finding the best value of C and the resulting correlation:

$$r_{IM} = .44$$
$$r_{Im} = .42$$
$$r_{Mm} = .66$$
$$\sigma_M = 16.59$$
$$\sigma_m = 16.65$$

When these values are substituted in formula (2), the value of C is found to be .79. This means that the best combination of time and unclassified errors is to add .79 of the time to the unclassified errors. This gives a correlation of .47; but if 1 is used in the formula instead of .79, the correlation is still .47. For our final score in this test, we shall use time and unclassified errors combined equally. The exact method of getting them will be to add together the scores in time and unclassified errors and then reduce to absolute percentiles by the same method as was used in Rational Learning.

(2) Analysis of Data.

There is a "present but low" positive correlation between the

criterion and repetitions and between the criterion and perseverative errors. The correlation of the criterion with each of the other factors is "marked." Subjects above the average in the former will tend to be above the average in each of the latter. Since the variability[27] is made the same in each kind of data by reducing to absolute percentiles, a unit-change in one test or factor will be accompanied by a like change in the other test or factor equal to the correlation of the two factors. Thus time and the criterion have a correlation of .42. Therefore every unit-change in one will be accompanied by .42 of a unit-change in the other. In like manner, each of the other factors may be compared with the criterion by reference to the correlations in Table XII.

Each factor in this test has much in common with the criterion. Unclassified errors have most and repetitions least. Table XII

TABLE XIII. Showing Correlations and Partial Correlations of the Factors in the Rational Learning Test (Modified).

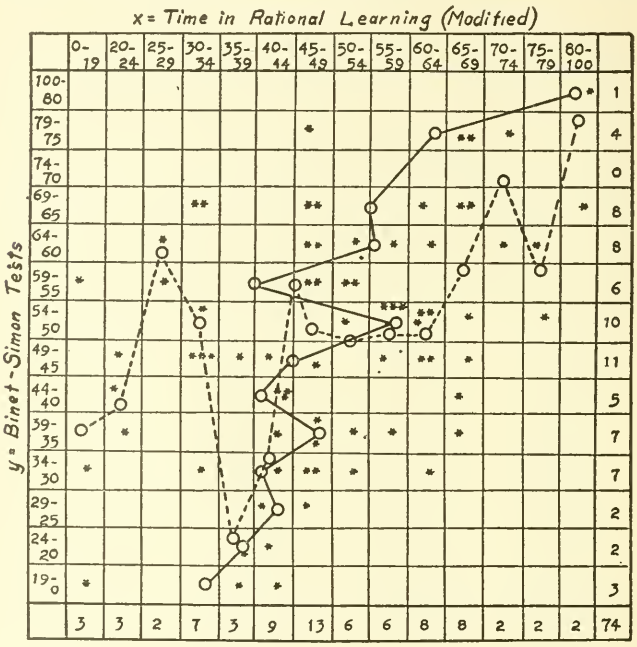
23	.60	24	.66	25	.61	26	.65	34	.77
23·4	.19	24·3	.39	25·3	.36	26·3	.45	34·2	.62
23·5	.33	24·5	.32	25·4	.01	26·4	.35	34·5	.53
23·6	.34	24·6	.38	25·6	.36	26·5	.45	34·6	.61
23·45	.20	24·35	.18	25·34	.05	26·34	.33	34·25	.47
23·46	.15	24·36	.23	25·36	.24	26.35	.38	34·26	.55
23·56	.21	24·56	.14	25·46	.07	26.45	.36	34·56	.45
23·456	.16	24·356	.06	25·346	.09	26·345	.34	34·256	.42
35	.67	36	.61	45	.92	46	.70	56	.60
35·2	.48	36·2	.36	45·2	.87	46·2	.48	56·2	.34
35·4	—.16	36·4	.16	45·3	.85	46·3	.45	56·3	.33
35·6	.48	36·5	.35	45·6	.87	46·5	.47	56·4	—.15
35·24	—.16	36·24	.10	45·23	.83	46·23	.34	56·23	.20
35·26	.41	36·25	.24	45·26	.85	46·25	.39	56·24	—.17
35·46	—.13	36·45	.14	45·36	.83	46·35	.35	56·34	—.13
35·246	—.14	36·245	.07	45·236	.82	46·235	.32	56·234	—.16

shows that time contains all that is common to repetitions and the criterion and nearly all that is common to perseverative errors and the criterion. Unclassified errors contain all that is common to the criterion and any one of the three factors, repetitions, logical errors and perseverative errors. Logical errors contain all that is common to repetitions and the criterion.

[27] The standard deviations for time, repetitions, unclassified errors, logical errors, and perseverative errors are 16.65, 16.48, 16.59, 16.68, and 16.35 respectively.

Table XIII shows that the factors have a "high" correlation with each other. Unclassified and logical errors have the highest. Since the correlation of these two factors is so high, neither of them can have very much that is not in the other.

TABLE XIV. Showing the Distribution of Subjects on the Basis of Time in Rational Learning (Modified) and the Binet-Simon Tests. Circles through which the broken line passes represent the means of the columns, and those through which the continuous line passes represent the means of the rows. Each asterisk represents a subject.



The several correlation-ratios for each of the factors with the criterion are as follows:

12 = .56 and .66
13 = .47 and .59
14 = .53 and .61
15 = .58 and .60
16 = .49 and .49

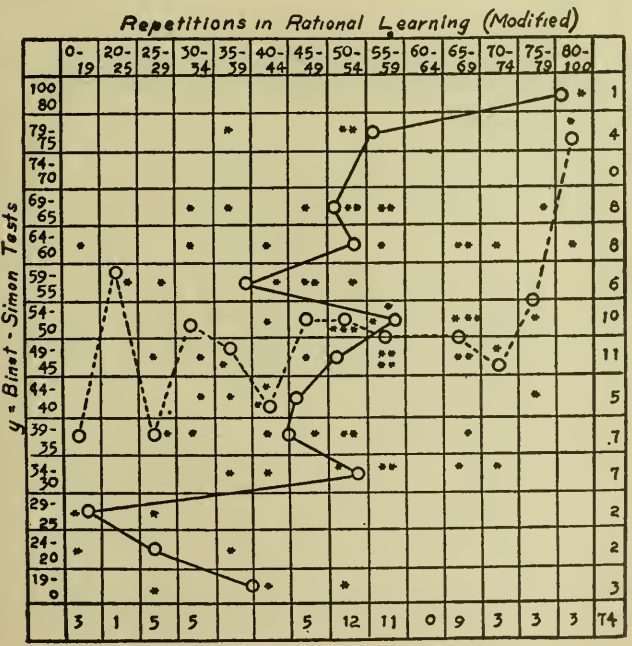
The Blakeman formula gives results as follows:

Tests 1 and 2, 2.36 and 3.24
Tests 1 and 3, 2.25 and 3.20
Tests 1 and 4, 1.87 and 2.69
Tests 1 and 5, 2.55 and 2.73
Tests 1 and 6, 2.19 and 2.19

All the correlations are non-linear except the last one, the criterion and perseverative errors.

Table XIV shows the correspondence between scores in time and the criterion. The Blakeman formula indicates that each of the regression lines is non-linear. The curve of the means of the rows shows very little change in x-values with an increase in y-values up to the fifty-fifth percentile, where the change is accelerated, probably, on account of the fact that the Binet-Simon

TABLE XV. Showing the Distribution of subjects on the Basis of Repetitions in Rational Learning (Modified) and the Binet-Simon Tests. Circles through which the broken line passes represent the means of the columns, and those through which the continuous line passes represent the means of the rows. Each asterisk represents a subject.



tests do not actually test the brightest subjects. The curve of the means of the columns has four distinct parts. As the x-values increase, the y-values decrease rapidly from the twenty-fifth to the thirty-fifth percentile, increase rapidly from the thirty-fifth to the forty-fifth percentile, remain about the same from the forty-fifth to the sixty-fifth percentile, and increase rapidly from the sixty-fifth percentile.

Table XV shows the correspondence between the scores in repetitions and the criterion. The regression line of the means of the columns may be considered linear according to the Blakeman test. The regression line of the means of the rows, however, is non-linear. The non-linearity is caused by the four cases in the second and third rows from the botton and the one case in the top row. The removal of these five cases will reduce the

TABLE XVI. Showing the Distribution of Subjects on the Basis of Unclassified Errors in Rational Learning (Modified) and the Binet-Simon Tests. Circles through which the broken line passes represent the means of the columns and those through which the continuous line passes represent the means of the rows. Each asterisk represents a subject.

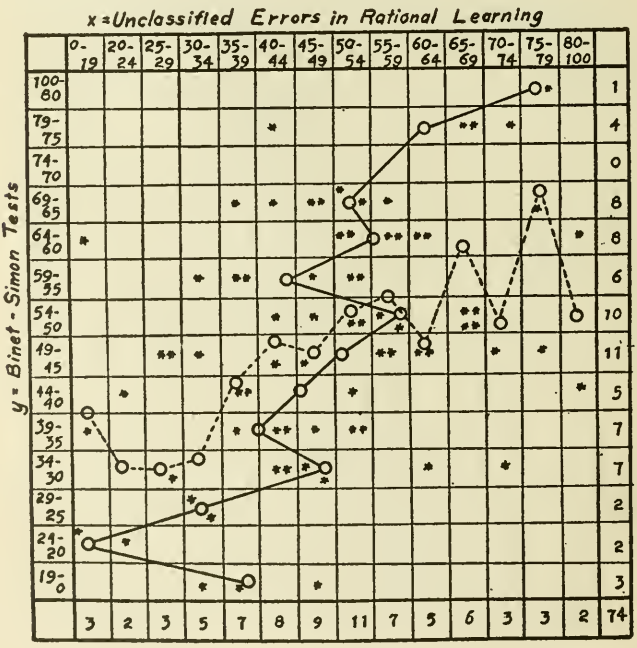
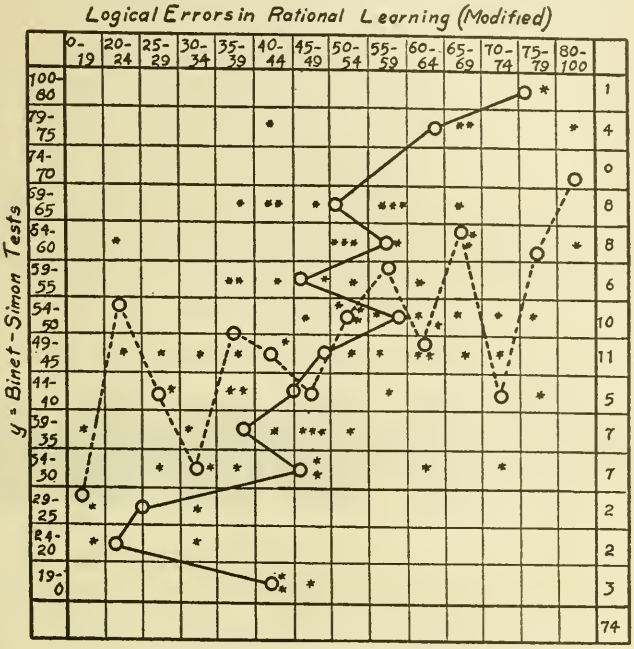


TABLE XVII. Showing the Distribution of Subjects on the Basis of Logical Errors in Rational Learning (Modified) and the Binet-Simon Tests. Circles through which the broken line passes represent the means of the columns and those through which the continuous line passes represent the means of the rows. Each asterisk represents a subject.



correlation-ratio from .59 to .34, provided the mean and standard deviation are not changed. Such a reduction in correlation-ratio will destroy the non-linearity.

Table XVI shows the correspondence between the scores in unclassified errors and the criterion. The regression line of the means of the columns is linear according to the test. The other regression line, however, is non-linear. The non-linearity can be eliminated by removing the two cases in the second row from the bottom. The same result can be obtained by assuming that these two cases fall on the median.

Table XVII shows the correspondence between the scores in logical errors and the criterion. Here both regression lines are slightly non-linear. The non-linearity, however, is due to a few cases and for that reason has no significance.

VII. Checker Puzzle

(1) Description of Test and Method of Scoring.

This may be called a checker puzzle test because of its similarity to the game of checkers. As far as the writer knows it was first used as a psychological experiment by Dr. Strong in the Jesup Psychological Laboratory of George Peabody College for Teachers. He also gives a suggestion for its use in the Psychological Bulletin.[28] The instructions to the subject and the method of scoring are different in many respects from those used in the Jesup Psychological Laboratory. They were devised by the writer, but, of course, reflect to a considerable degree those with which he was already acquainted.

Fig. II. Showing Apparatus Used in the Checker Puzzle Test.

The subject is given a card on which are seven circles as shown in Figure II. He is also given three red and three black checkers. The instructions to the subject are as follows:

"You are here given a row of seven circles. The three at the left are red, the three at the right are black, and the middle one is black on the right half and red on the left half. You are also given three red and three black checkers. You are to place the three red checkers on the three black circles and the three black checkers on the three red circles.

"Your problem is to get the three red checkers on the three red circles and the three black checkers on the three black circles. You must move but one checker at a time, jump only one at a time, and never move backwards. When you are blocked so that you cannot move farther, set the checkers back to the starting point and begin anew. When you can go through the problem three times in succession without any errors, we will consider

[28] Strong, E. K., Jr., The Learning Process, "*Psychol. Bull.*, 1918, XV, 328 ff.

it learned. Keep in mind that your results are being judged (1) by the time spent, (2) by the number of attempts (each time you begin counting an attempt, whether you are successful or not), (3) by the number of successful solutions required for the learning."

The subject sat at one side of the table and the experimenter at the other. The latter kept an accurate account of the time, the number of attempts, and the number of successful solutions.

The results are found in Table XVIII.

TABLE XVIII. Showing the Number of Minutes, the Number of Attempts, the Number of Solutions, and the Percentile Rank for Each Kind of Data in the Checker Puzzle Test.

	Score in			Percentile Rank in				Score in			Percentile Rank in		
Subject	T	A	S	T	A	S	Subject	T	A	S	T	A	S
1	9	11	4	74	15	78	38	19	30	12	53	39	25
2	16	15	6	58	63	59	39	8	7	5	81	86	69
3	17	23	9	56	51	40	40	9	8	6	74	82	59
4	26	14	6	41	65	59	41	14	16	6	61	61	59
5	12	20	8	67	54	46	42	13	17	6	64	59	59
6	22	18	6	49	58	59	43	18	16	10	55	61	35
7	26	14	3	41	65	86	44	19	30	6	53	39	59
8	31	28	9	35	43	40	45	13	12	8	64	70	46
9	16	18	8	58	58	46	46	25	27	8	45	44	46
10	29	24	7	38	48	52	47	30	25	7	37	46	52
11	29	29	6	38	42	59	48	58	57	9	14	18	40
12	11	14	5	69	65	69	49	24	30	6	45	39	59
13	19	32	6	53	36	59	50	52	49	11	19	25	31
14	10	12	6	70	70	59	51	52	49	11	19	25	31
15	21	25	15	51	46	52	52	12	19	7	67	56	52
16	29	23	6	38	51	59	53	21	30	8	51	39	46
17	23	24	9	47	48	40	54	24	25	5	45	46	69
18	20	10	8	52	77	46	55	26	37	11	41	34	31
19	19	23	7	53	51	52	56	41	52	14	23	21	0
20	35	45	9	33	28	40	57	8	13	5	81	67	69
21	13	9	6	64	79	59	58	14	16	5	61	61	69
22	22	30	12	49	39	25	59	25	33	13	43	35	16
23	9	23	6	74	51	59	60	35	41	10	33	30	35
24	35	23	8	33	51	46	61	25	21	8	43	53	46
25	36	46	13	29	27	16	62	16	19	5	58	56	69
26	6	12	8	86	70	46	63	37	29	8	25	42	46
27	12	19	4	67	56	78	64	19	16	4	53	61	78
28	17	28	8	56	43	46	65	15	39	8	60	32	46
29	30	23	6	37	51	59	66	9	21	5	69	53	69
30	13	12	8	64	70	46	67	28	12	10	40	70	35
31	36	31	10	29	37	35	68	12	18	8	67	58	46
32	24	25	6	45	46	59	69	36	62	11	29	14	31
33	23	20	9	47	54	40	70	23	32	9	47	36	40
34	36	38	7	29	33	52	71	62	80	12	0	0	25
35	21	23	9	51	51	40	72	15	13	4	60	67	78
36	23	27	10	47	44	35	73	16	17	7	58	59	52
37	21	40	12	51	31	25	74	33	23	6	35	51	59

TABLE XIX. Showing Correlations and Partial Correlations of Each Factor* of the Checker Puzzle with the Binet-Simon Test and with Each Other.

12	.26	13	.18	14	.20
12·3	.18	13·2	.03	14·3	.06
12·4	.19	13·4	.05	14·2	.08
12·34	.18	13·24	—.03	14·23	.06
23	.62	24	.51	34	.76
23·4	.41	24·3	.08	34·2	.65

*For the sake of brevity the factors will be designated as follows:

1. Binet-Simon Tests
2. Solutions
3. Attempts
4. Time.

Since $r_{14\cdot23} = .06$, factor 4 has nothing in common with the criterion that is not contained in factors 2 and 3. In like manner, since $r_{13\cdot2} = .03$, factor 3 has nothing in common with the criterion that is not contained in factor 2. Nothing will be lost, therefore, as far as the criterion is concerned, by discarding time and attempts from the final score. For the present purpose, then, we shall use only the number of solutions as the final score. The formula for multiple correlation shows that it is not possible by combining the factors to get a higher correlation with the criterion than .27. The range of solutions is so small that the scores are bunched considerably. This probably affects the correlation somewhat, but there is no way to remedy it. It cannot be raised by using the other factors in any way. The present method has for its purpose to get the highest correlation with the criterion.

(2) Analysis of Data.

There is a "present but low" positive correlation between the criterion and time and between the criterion and solutions. The correlation is negligible, however, between the criterion and the number of attempts. Since the variability[29] is the same in each kind of data, each factor may be directly compared with the criterion by reference to Table XIX. Thus, for every unit-change in solutions there will be a like change of .26 of a unit in the criterion, and vice versa.

[29] The standard deviations for time, attempts, and solutions are 16.57, 16.54, and 16.1 respectively.

The factors, solutions and time, have something in common with the criterion. Solutions have most and contain elements not contained in either of the other factors. Time contains nothing with respect to the criterion that is not contained also in solutions. Each of the factors is composed largely of elements not found in the criterion. Solutions and attempts have a high correlation with each other as have time and attempts. The correlation of solutions and time is marked.

The correlation-ratios for each of the factors with the criterion are as follows:

12 = .49 and .54
13 = .30 and .42
14 = .40 and .44

These values substituted in the Blakeman formula give results as follows:

Tests 1 and 2, 2.71 and 3.05
Tests 1 and 3, 1.52 and 2.42
Tests 1 and 4, 2.21 and 2.50

According to our test, the correlation of the criterion and solutions is non-linear. The other two correlations are linear. A table will be constructed to show the regression lines in the correlation of solutions and the Binet-Simon tests. This will be postponed, however, until the final scores are analyzed.

VIII. The Tait Labyrinth Puzzle

(1) Description of Test and Method of Scoring.

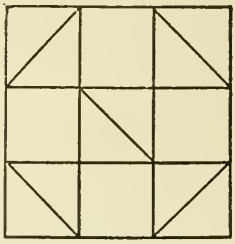
In this test the subject was given a figure of the Tait Labyrinth Puzzle and a copy of the instructions. Freeman[30] has given suggestions as to its use. Lindley[31] also used it in his "Study of Puzzles." The figure and instructions are here given.

"You have before you a figure that can be drawn without lifting the pencil from the paper and without retracing. Your problem is to draw the figure without lifting the pencil from the paper and without retracing. As soon as you are ready you may begin on this blank sheet of paper. You may keep the figure

[30] Freeman, F. N., *Experimental Education*, 1916, 36 ff.
[31] Lindley, E. H., Study of Puzzles, *Amer. J. of Psychol.*, *8*, 430 ff.

before you and refer to it during the drawing if you wish. No attention will be given to the technical excellence of the drawing. If you fail in the first attempt, take another sheet of paper and try it again. Continue until you have made the figure three times

Fig. III. Showing the Tait Labyrinth Puzzle.



in succession without any errors. You are to be judged by the number of trials required for the learning and by the number of minutes used."

When the subject started, the time was noted and then noted again when the problem was complete. This time included that used in reading the directions as well as that used in solving the problem. It was thought necessary to include the time used in reading the directions, since so many will trace the pencil in the air over the figure before trying to draw it on paper.

If we designate the number of trials by 2 and the number of minutes by 3, the correlations are as given in Table XXI.

The best possible combination of time and trials gives a correlation of .30 with the Binet-Simon tests. Since $r_{13 \cdot 2}$ gives a correlation of .01, there is nothing common to time and the criterion that is not contained in trials. Therefore the final scores for comparison with the criterion will consist of the percentile ranks in number of trials.

(2) Analysis of Data.

There is a "present but low" positive correlation between the criterion and number of trials. The correlation of the criterion with the number of minutes, however, is "negligible." Since the

TABLE XX. Showing the Number of Trials, the Number of minutes, and the Percentile Rank in Each Kind of Data in Tait Labyrinth Puzzle.

Subject	Score in		Per. Rank in		Subject	Score in		Per. Rank in	
	Trials	Time	Trials	Time		Trials	Time	Trials	Time
1	11	6	43	66	38	34	52	14	14
2	19	24	31	33	39	5	11	67	50
3	6	8	59	59	40	17	10	35	54
4	5	10	67	54	41	6	11	59	50
5	6	6	59	66	42	5	7	67	63
6	12	13	42	46	43	6	10	59	54
7	14	14	37	45	44	25	31	23	21
8	22	21	27	38	45	7	12	57	48
9	5	6	67	66	46	10	11	46	50
10	25	24	23	33	47	9	7	50	63
11	5	22	67	37	48	17	29	35	25
12	9	10	50	54	49	9	18	50	21
13	3	13	82	46	50	14	26	37	28
14	7	7	57	63	51	5	9	67	57
15	25	24	23	33	52	13	23	40	35
16	13	25	40	30	53	10	8	46	59
17	17	30	35	23	54	12	17	42	42
18	11	4	43	72	55	18	13	33	46
19	6	8	59	59	56	18	67	33	0
20	10	11	46	50	57	25	17	23	42
21	22	19	28	40	58	10	10	46	54
22	9	4	50	72	59	13	15	40	43
23	11	19	43	40	60	8	7	53	63
24	8	7	53	63	61	5	11	67	50
25	8	10	53	54	62	4	4	76	72
26	20	26	31	28	63	5	15	67	43
27	5	8	67	59	64	7	3	57	78
28	8	2	53	84	65	8	23	53	35
29	3	5	82	68	66	8	33	53	18
30	9	4	50	72	67	10	12	46	48
31	13	21	40	38	68	5	3	67	78
32	3	8	82	59	69	10	11	46	50
33	14	23	37	35	70	6	2	59	84
34	9	24	50	33	71	38	14	0	45
35	5	10	67	54	72	4	10	76	54
36	6	9	59	57	73	5	4	67	72
37	13	8	40	59	74	8	19	53	40

TABLE XXI. Showing the Correlations and Partial Correlations of Each Factor in the Tait Labyrinth Puzzle with the Binet-Simon Tests.

12	.30
12·3	.26
13	.17
13·2	.01
23	.55

variability[32] is the same in each kind of data, each factor may be compared directly with the criterion by reference to Table XXI. Thus, for every unit change in number of trials there will be a like change of .30 of a unit in the criterion and vice versa.

It has already been noted that time contains nothing with respect to the criterion that is not contained in number of repetitions. Trials and minutes show a "marked" correlation with each other.

The correlation-ratios of the factors of this test with the criterion are:

$$12 = .46 \text{ and } .54$$
$$13 = .33 \text{ and } .44$$

When these values are substituted in Blakeman's formula, results as follows are obtained:

For tests 1 and 2, 2.22 and 2.86
For tests 1 and 3, 1.80 and 2.59

One of the regression lines in each correlation is linear and the other non-linear. Table XXII shows the correspondence between time and the criterion. The regression line of the means of the columns is relatively linear and shows an increase in y-values with an increase in x-values from the lowest to the highest percentile. The regression line of the means of the rows is relatively non-linear. It shows a rapid increase in x-values with an increase in y-values from the twentieth to the fortieth percentile. The x-values change very little until the sixtieth percentile is reached and then the increase is again rapid.

Since the percentile ranks in trials are used as the final scores, the table showing the correspondence between this factor and the criterion will be postponed until the next section.

IX. Intercorrelations

(1) Tests Analyzed in the Light of the Criterion.

We shall first analyze the scores in the light of the criterion. The final scores are obtained in Rational Learning by combining repetitions and perseverative errors equally, in Rational Learn-

[32] The standard deviations for trials and time are 16.33 and 16.58 respectively.

TABLE XXII. Showing the Distribution of Subjects on the Basis of Time in the Tait Labyrinth Puzzle and the Binet-Simon Tests. Circles through which the broken line passes represent the means of the columns and those through which the continuous line passes represent the means of the rows. Each asterisk represents a subject.

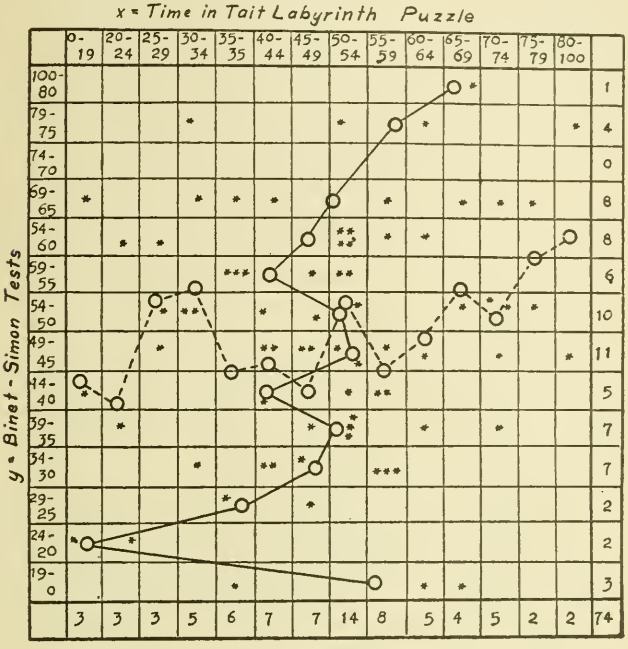


TABLE XXIII. Showing the Correlations and Partial Correlations for the Final Scores* with the Binet-Simon Tests.*

12	.33	13	.47	14	.26	15	.30
12·3	.20	13·2	.40	14·2	.17	15·2	.19
12·4	.27	13·4	.43	14·3	.13	15·3	.25
12·5	.24	13·5	.45	14·5	.19	15·4	.25
12·34	.18	13·24	.38	14·23	.09	15·23	.19
12·35	.11	13·25	.40	14·25	.14	15·24	.17
12·45	.21	13·45	.42	14·35	.07	15·34	.22
12·345	.11	13·245	.38	14·235	.06	15·234	.18

* The numbers have the following meaning:

1. Binet-Simon Tests.
2. Rational Learning.
3. Rational Learning (Modified).
4. Checker Puzzle.
5. Tait Labyrinth Puzzle.

ing (Modified) by combining minutes and unclassified errors equally, in the Checker Puzzle by taking the solutions, and in the Tait Labyrinth Puzzle by taking the number of trials.

(a) Rational Learning. There is a "present but low" positive correlation between the final scores in Rational Learning and the criterion. The correlation is partly due to elements found also in Rational Learning (Modified), found to a less extent in the Tait Labyrinth Puzzle, and to a still less extent in the Checker Puzzle. The correlation is significant when the common elements in any one of the other three tests are removed, but when the common elements found also in all the other three tests are removed, the correlation is no longer significant. In other words, everything common to the criterion and Rational Learning is found in the other three tests.

The correlation ratios for The Binet-Simon tests and Rational Learning are .47 and .51. These values substituted in Blakeman's formula give 2.13 and 2.47, which indicate that for all practical purposes the correlation is linear.

Rational Learning seems to measure some mental functions not detected by the Binet-Simon tests. The first that may be mentioned is that of being able to attack and solve a problem without getting confused. In support of this statement some special cases are cited. Subject 6 scores "high" in each factor of Rational Learning, but "low" in the criterion. She grasped the situation quickly and completed the learning with only 23 errors. Subject 9 scores "high" in unclassified, logical, and perseverative errors, but "low" in the criterion. She made only 17 errors in the first repetition and finished with a total of 21. We have altogether nine subjects who score "low" in the criterion and "high" in one or more factors in Rational Learning. An examination of the individual records shows that in every case the learning was completed without confusion or distraction.

A second mental function that Rational Learning seems to test better than does the criterion is the ability to give attention longer and to more elements than is usually required in the latter tests. Subject 26 illustrates this point fairly well. She scores

"high" in the criterion and "low" in repetitions and perseverative errors. She has 88 unclassified errors altogether and 70 of these were made in the first two repetitions. The other 18 are distributed from the third to the eighth inclusive. The record indicates that the learning was almost complete in the third repetition, in which only two errors were made, yet five more repetitions were required. Subjects 34 and 42 have records similar to that of 26; that is, they have a few errors distributed over several repetitions, a condition which indicates a lack of attention.

A third mental function or process measured better by Rational Learning than by the criterion is the kind of attack. Some subjects read the instructions and make sure that every point is understood before beginning. Others read them in a careless way and jump into the problem without knowing just what is to be done. Subject 2 illustrates the later method. She made 77 unclassified and 44 logical errors in the first two repetitions. Subject 72 made 5 of her 8 perseverative errors in the first repetition. This indicates that the instructions were not fully understood.

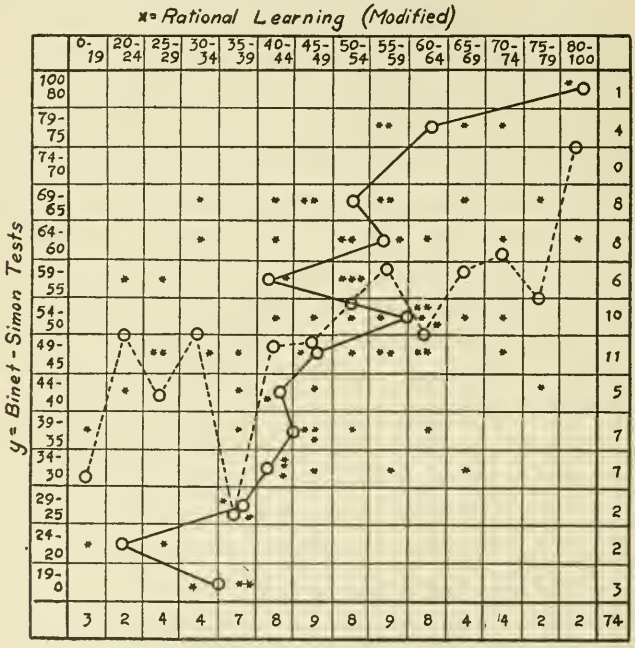
The fourth and last mental function that seems to be especially well brought out by Rational Learning is the speed of the subject. This may be illustrated by subject 72. She scores "high" in the criterion and "low" in time and perseverative errors. She is a mature woman who goes at everything slowly and deliberately. Subject 40 also is a good example. She worked very slowly and deliberately, thus making a "high" score in repetitions.

(b) Rational Learning (Modified). The correlation of Rational Learning (Modified) and the criterion is "marked." The elements common to the two tests are found to a slight extent in each of the other three tests. The correlation of the third order shows that Rational Learning (Modified) has elements common to the criterion not found in any one of the other three tests or in all of them combined. This means that Rational Learning (Modified) contains elements not found in the other tests.

The correlation-ratios for Rational Learning (Modified) and the criterion are .60 and .63. These values substituted in the

Blakeman formula give 2.37 and 2.69, indicating that one regression line is linear and the other non-linear. Table XXIV shows the actual regression lines. The line joining the means of the rows is relatively linear and the line joining the means of the columns is relatively non-linear. The non-linearity would be eliminated if the average of the seven cases in the fifth column from the left were 50 instead of 26. The number of cases in each row and column is too small for the non-linearity to have any significance when the curve does not take any well defined shape.

TABLE XXIV. Showing the Distribution of Subjects on the Basis of Rational learning (Modified) and the Binet-Simon Tests. Circles through which the broken line passes represent the means of the columns and those through which the continuous line passes represent the means of the rows. Each asterisk represents a subject.



In Rational Learning, repetitions and perseverative errors are the significant factors. In Rational Learning (Modified), however, the significant factors are time and unclassified errors. The cause of this difference is interesting and can be stated only

on a priori grounds. In the former, repetitions are more significant than time and contain everything in time with respect to the criterion. The reverse, however, is the case with Rational Learning (Modified). Time includes all that is in repetitions. The writer is of the opinion that the experimenter, in calling out the numbers, controls the speed of the subject to some extent. He enters into the situation in a different way from what he does when he stands back and records the responses. The writer has found that when a subject is naming words, the speed is checked if the words are recorded in plain view of the subject.

The next problem is to try to answer why perseverative errors in Rational Learning and unclassified errors in Rational Learning (Modified) are the significant factor. This also can be stated only on a priori grounds. It is probable that in the latter experiment space perception makes it easier to avoid perseverative errors than in the former. The tendency seems to be to go from one end of a row to the other and to skip about here and there less than in Rational Learning.

An analysis of individual cases indicates that Rational Learning (Modified) tests the same factors as Rational Learning. First, it tests the subject's ability to work for a period of time without confusion or distraction better than the criterion does. In support of this some special cases are cited. Subject 6 scores "low" in the criterion and "high" in time, repetitions, and unclassified errors. The data shows that she was able to concentrate her attention and learn the problem without confusion. Subject 22 scores "low" in the criterion and "high" in time, repetitions, and perseverative errors. She learned this test quickly and was able to avoid confusion and distraction.

Second, Rational Learning (Modified) tests a subject's ability to give attention longer and to a more complex situation than that usually required by the criterion. Subject 11 illustrates this point. She scores "high" in the criterion and "low" in repetitions. In five of the repetitions only one error was made for each. Certainly close attention would have cut down the num-

ber. Subject 49 has a "high" score in the criterion and a "low" one in repetitions and perseverative errors. The "low" score in repetitions of this subject also is caused by a lack of attention, as was that of subject 11. This conclusion is based on the fact that she made from zero to three errors in each repetition from the sixth to the thirteenth. Subject 61 scored "high" in the criterion and "low" in repetitions and perseverative errors. She made only one error in the third repetition, yet she required thirteen repetitions to complete the learning. The greatest number of errors made in any repetition after the second is three. This too probably shows a lack of attention to the correct numbers.

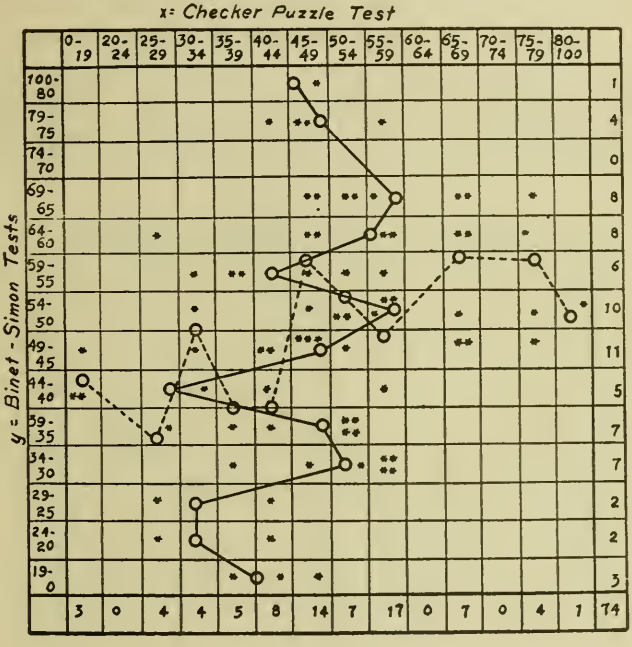
In the third place, Rational Learning (Modified) is better for detecting the kind of attack than is the criterion. It is possible to determine whether the subject approaches the problem with that deliberate method which indicates that he is sure of what is to be done, or approaches it in that method characteristic of the person who gets an inkling of what is to be done and then begins in a kind of hit-and-miss sort of way.

The fourth mental function that is revealed in this test is the speed of the subject. Here we have reference to the procedure after the instructions have been read and the subject has begun. This is illustrated by subject 66. He scores "high" in the criterion and "low" in time. The 'low" score in the latter is clearly due to the slow, deliberate method of work. Subject 72 also scores "high" in the criterion and low in time. Her scores are almost identical with those of subject 66. She is a mature woman who worked very slowly and deliberately.

(c) Checker Puzzle. There is a "present but low" positive correlation between the scores in the Checker Puzzle and the criterion. The correlation is partly explained by elements found also in each of the other tests. This test as far as the criterion is concerned is most like Rational Learning (Modified and least like the Tait Labyrinth Puzzle. The correlation is not significant when the common elements found also in the other tests are removed.

The correlation-ratios for the criterion and the Checker Puzzle are .49 and .54. These values substituted in Blakeman's formula give 2.71 and 3.05, indicating that both of the regression lines are non-linear. Table XXV shows the two regression lines. If they were smoothed they would be nearly straight and show very little correlation. In other words the high correlation-ratio is to a great extent due to the fluctuation of the means of the rows and the means of the columns.

TABLE XXV. Showing the Distribution of Subjects on the Basis of the Checker Puzzle and the Binet-Simon Tests. Circles through which the broken line passes represent the means of the columns and those through which the continuous line passes represent the means of the rows. Each asterisk represents a subject.



Analysis of the individual cases shows no mental functions tested by the Checker Puzzle that are not also tested by the criterion. This may be due to the fact that the responses of the subjects cannot be recorded so exactly as in the other tests. Partial correlations of the third order show that this test contains nothing

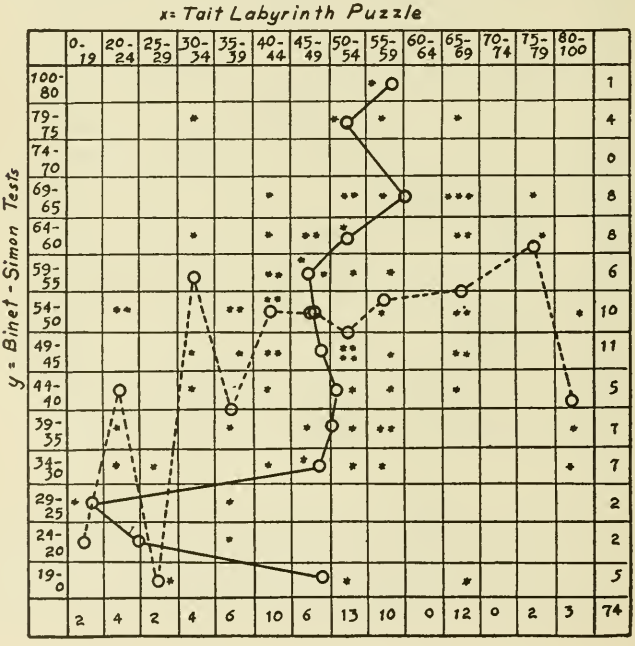
with respect to the criterion that is not contained in the two Rational Learning Tests.

(d) Tait Labyrinth Puzzle. The correlation between the criterion and the Tait Labyrinth Puzzle is positive and "present but low." The correlation is partly due to elements found also in Rational Learning, and to a less extent to elements found in Rational Learning (Modified) and the Checker Puzzle. The correlation is barely significant when the elements common to all three tests are removed.

The individual cases reveal no mental functions tested by the Tait Labyrinth Puzzle that are not tested by the criterion.

The correlation-ratios for the criterion and the Tait Labyrinth Puzzle are .46 and .54. These values substituted in the Blakeman formula give 2.22 and 2.86, indicating that one regression

TABLE XXVI. Showing the Distribution of Subjects on the Basis of the Tait Labyrinth Puzzle and the Binet-Simon Test. Circles through which the broken line passes represent the means of the columns and those through which the continuous line passes represent the means of the rows.



line is linear and the other non-linear. The line joining the means of the rows is linear and the other is non-linear. Table XXVI shows the two regression lines.

(2) Interrelation of Tests Scored in the Light of the Criterion. The four tests are here compared with each other as they are scored in the light of the criterion.

TABLE XXVII. Showing the Correlations and Partial Correlations of the Four Tests, When Scored in the Light of the Criterion.*

23	.36	24	.32	25	.41
23·4	.29	24·3	.23	25·3	.38
23·5	.32	24·5	.24	25·4	.36
23·45	.27	24·35	.17	25·34	.35
34	.32	35	.18	45	.26
34·2	.23	35·2	.04	45·2	.15
34·5	.28	35·4	.11	45·3	.22
34·25	.22	35·24	.01	45·23	.14

* The numbers have the same meanings as in Table XXIII.

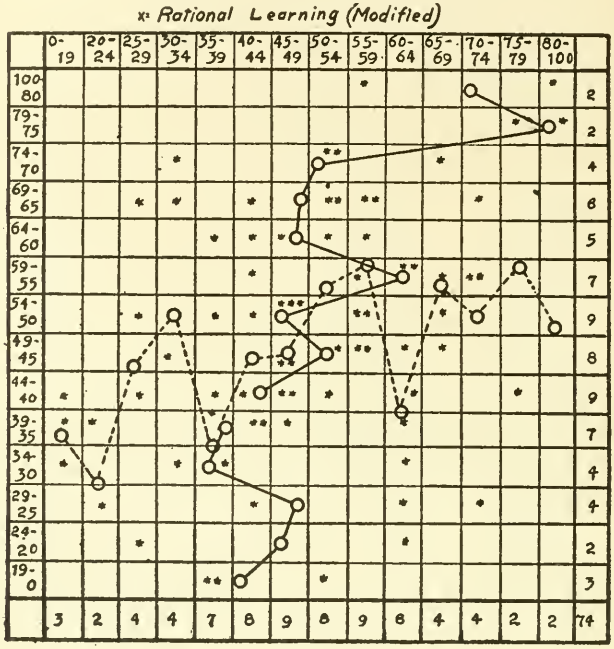
Rational Learning has a "present but low" positive correlation with Rational Learning (Modified) and with the Checker Puzzle. It has a "marked" correlation with the Tait Labyrinth Puzzle. There are elements common to Rational Learning and Rational Learning (Modified) that are not found in the Checker Puzzle and the Tait Labyrinth Puzzle. In like manner there are elements common to Rational Learning and the Tait Labyrinth Puzzle that are not found in the other two tests. The correlation of Rational Learning with the other three tests combined is .53.

The correlation of Rational Learning (Modified) and the Checker Puzzle is "present but low." The correlation with the Tait Labyrinth Puzzle is barely significant. There are elements common to Rational Learning (Modified) and the Checker Puzzle not found in the other two tests. The correlation of this test with the other three combined is .48.

The Checker Puzzle has a "present but low" positive correlation with the Tait Labyrinth Puzzle. The correlation is mostly due to elements found also in Rational Learning and Rational Learning (Modified). The correlation with the other three tests combined is .41.

The Tait Labyrinth Puzzle has a correlation with the other

TABLE XXVIII. Showing the Distribution of Subjects on the Basis of Rational Learning and Rational Learning (Modified). Circles through which the broken line passes represent the means of the columns and those through which the continuous line passes represent the means of the rows. Each asterisk represents a subject.



three tests combined of .43. The multiple correlations may be summarized as follows:

$$R_{2(345)} = .53$$
$$R_{3(245)} = .48$$
$$R_{4(235)} = .41$$
$$R_{5(234)} = .43$$

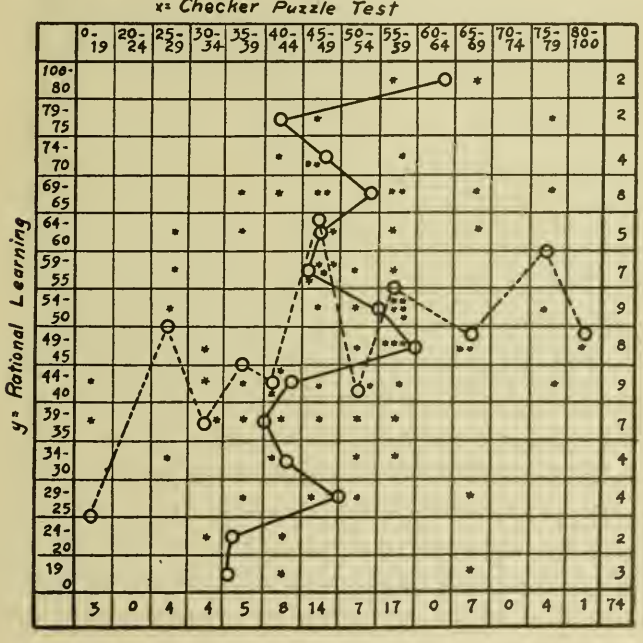
After allowance is made for errors and non-linearity, it seems safe to conclude that each of these tests contains elements that are not found in any of the others. The correlation-ratios for the inter-relations are as follows:

23 = .51 and .57
24 = .46 and .60
25 = .47 and .51
34 = .51 and .58
35 = .35 and .51
45 = .43 and .46

When these values are substituted in the Blakeman formula, results obtained are as follows:

Tests 2 and 3, 2.30 and 2.82
Tests 2 and 4, 2.02 and 3.23
Tests 2 and 5, 1.46 and 1.93
Tests 3 and 4, 2.01 and 3.08
Tests 3 and 5, 1.91 and 3.04
Tests 4 and 5, 2.18 and 2.42

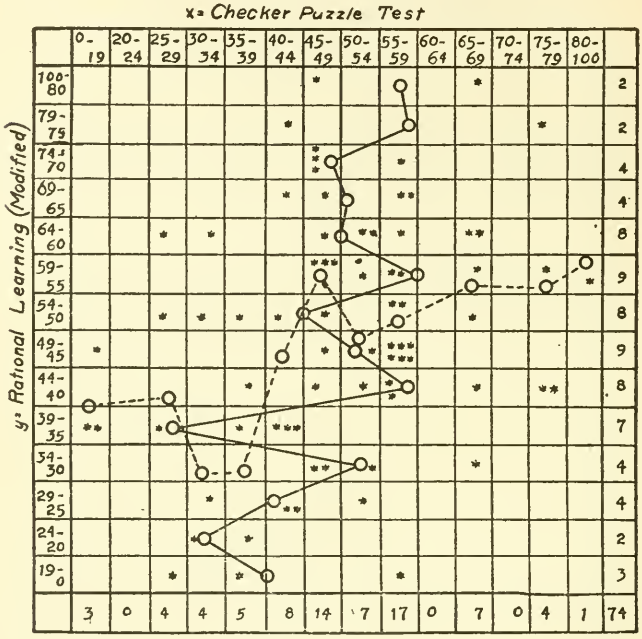
TABLE XXIX. Showing the Distribution of Subjects on the Basis of Rational Learning and the Checker Puzzle. Circles through which the broken line passes represent the means of the columns and those through which the continuous line passes represent the means of the rows. Each asterisk represents a subject.



Four of the six correlations are non-linear according to the Blakeman test. The correlation of tests 2 and 5 and of tests 4 and 5 may be considered linear. Tables showing the lines of the means of the columns and the means of the rows will now be constructed for the non-linear correlations.

Table XXVIII shows the correspondence between the scores in Rational Learning and Rational Learning (Modified). The lines appear linear from the thirteenth to the sixteenth percentiles

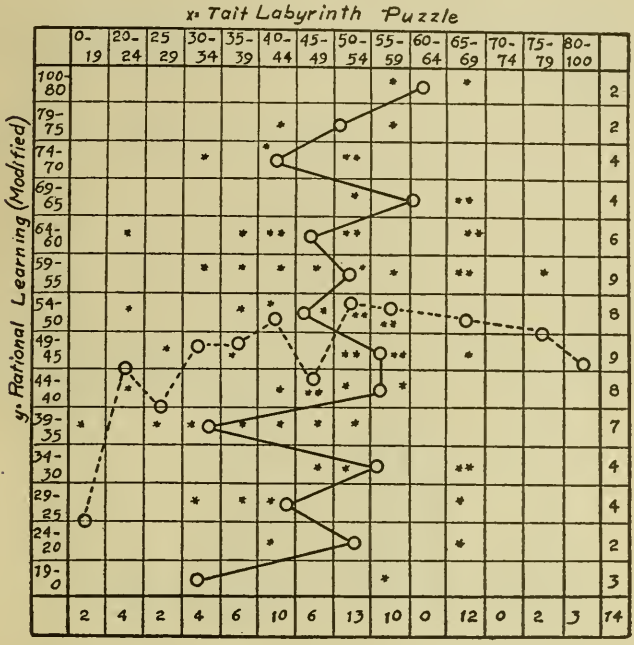
TABLE XXX. Showing the Distribution of Subjects on the Basis of Rational Learning (Modified) and the Checker Puzzle. Circles through which the broken line passes represent the means of the columns and those through which the continuous line passes represent the means of the rows. Each asterisk represents a subject.



and show a high correlation within these limits. Outside of these limits, however, the fluctuations are marked and consequently the correlation-ratio becomes larger than the correlation.

Table XXIX shows the correspondence between the scores in Rational Learning and Checker Puzzle. These regression lines when smoothed will be approximately straight and will show a

TABLE XXXI. Showing the Distribution of Subjects on the Basis of Rational Learning (Modified) and the Tait Labyrinth Puzzle. Circles through which the broken line passes represent the means of the columns and those through which the continuous line passes represent the means of the rows. Each asterisk represents a subject.



low positive correlation. An increase in ability to do the Checker Puzzle shows an increase in ability to do Rational Learning. The reverse, however, is not so true. An increase in ability to perform Rational Learning does not show very much change in ability to perform the Checker Puzzle.

Table XXX shows the correspondence between the scores in Rational Learning (Modified) and the Checker Puzzle. The line of the means of the rows smoothed will be approximately straight, but the other regression line will be far from straight. There seems to be closer agreement between the two sets of scores in the upper quartiles than in the lower.

Table XXXI shows the agreement of scores in Rational Learning (Modified) and the Tait Labyrinth Puzzle. An increase in ability in Rational Learning (Modified) is accompanied by an

increase in ability in the Tait Labyrinth Puzzle. Here again, the reverse is not true. After the fiftieth percentile, an increase in ability in the Tait Labyrinth Puzzle is accompanied by a decrease in ability in Rational Learning (Modified).

(3) Interrelation of Tests Scored by Combining the Factors Equally.

The next step will be to analyze the four tests when scored by combining all the factors equally. The final scores in Rational Learning are obtained by adding together the percentile ranks in time, repetitions, unclassified, logical and perseverative errors and then reducing to absolute percentiles by use of Rugg's table. The final scores in the other tests are found in a similar manner.

TABLE XXXII. Showing the Correlations and Partial Correlations, When Scored by Combining All the Factors Equally.*

23	.33	24	.18	25	.44
23·4	.29	24·3	.06	25·3	.37
23·5	.21	24·5	.08	25·4	.41
23·45	.20	24·35	.01	25·34	.37
34	.38	35	.33	45	.26
34·2	.34	35·2	.21	45·2	.20
34·5	.32	35·4	.26	45·3	.15
34·25	.31	35·24	.15	45·23	.14

* The numbers have the same meaning as in Table XXIII.

Rational Learning has something in common with each of the other tests. It is most like the Tait Labyrinth Puzzle and least like the Checker Puzzle. The correlation with the former is "marked" and with the latter is barely significant. Everything in the Checker Puzzle common to Rational Learning is also found in the Tait Labyrinth Puzzle, and in Rational Learning (Modified). There are elements common to Rational Learning and Rational Learning (Modified) that are not found in the Checker Puzzle and the Tait Labyrinth Puzzle. In like manner there are elements common to Rational Learning and the Tait Labyrinth Puzzle that are not found in the other two tests. The correlation of Rational Learning with the other three tests is .48.

Rational Learning (Modified) has a "present but low" positive correlation with the Checker Puzzle and with the Tait Laby-

rinth Puzzle. The correlation with the other three tests combined is .48. This means that there is much in this test common to the other three tests and much that is not found in them.

The Checker Puzzle test has a low correlation with the Tait Labyrinth Puzzle. It correlates with the other three tests combined to the extent of .40. The Tait Labyrinth Puzzle has a correlation with the three remaining tests of .49. The multiple correlations may be summarized as follows:

$$R_{2(345)} = .48$$
$$R_{3(245)} = .48$$
$$R_{4(235)} = .40$$
$$R_{5(234)} = .49$$

We may safely conclude that each of these tests contains much that is not found in any of the other tests. The correlation-ratios are as follows:

$$23 = .45 \text{ and } .49$$
$$24 = .33 \text{ and } .54$$
$$25 = .50 \text{ and } .61$$
$$34 = .53 \text{ and } .59$$
$$35 = .47 \text{ and } .49$$
$$45 = .40 \text{ and } .47$$

When these values are substituted in Blakeman's formula, the following results are obtained:

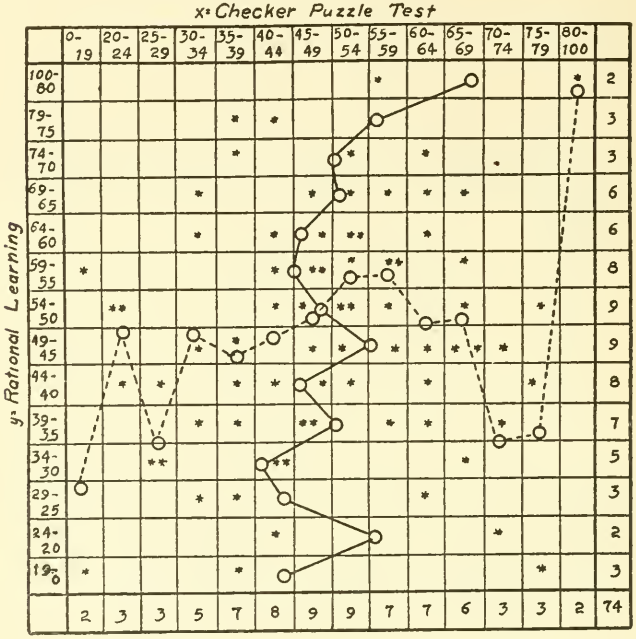
For tests 2 and 3, 1.95 and 2.30
For tests 2 and 4, 1.76 and 3.24
For tests 2 and 5, 1.51 and 2.61
For tests 3 and 4, 2.35 and 2.88
For tests 3 and 5, 2.13 and 2.30
For tests 4 and 5, 1.96 and 2.49

According to the Blakeman test three of the correlations are linear and three are non-linear. The correlation of Rational Learning with Rational Learning (Modified) is linear, but with the Checker Puzzle and the Tait Labyrinth Puzzle it is non-linear. The correlation of Rational Learning (Modified) with the Checker Puzzle is non-linear, but with the Tait Labyrinth

Puzzle it is linear. The correlation of the Checker Puzzle with the Tait Labyrinth Puzzle is linear.

Table XXXIII shows the correspondence between scores in Rational Learning and the Checker Puzzle. The line joining the means of the rows shows that as ability in Rational Learning

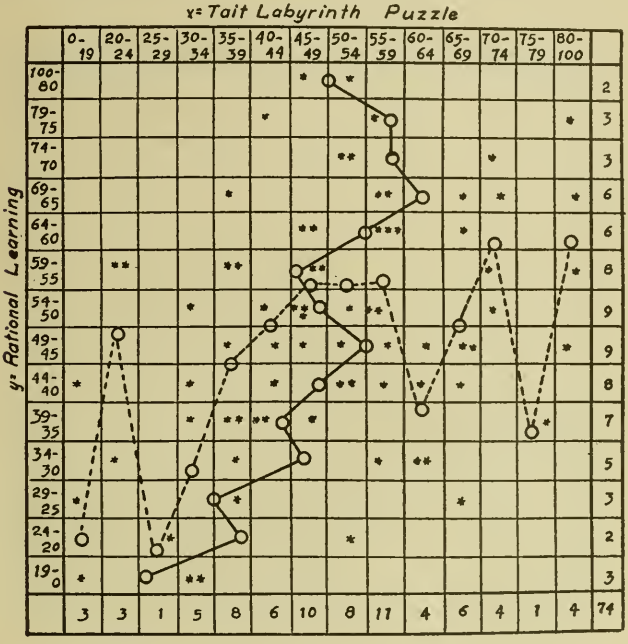
TABLE XXXIII. Showing Distribution of Subjects on the Basis of Rational Learning and the Checker Puzzle. Circles through which the broken line passes represent the means of the columns and those through which the continuous line passes represent the means of the rows. Each asterisk represents a subject.



increases there is not much change in ability in the Checker Puzzle until the seventieth percentile is reached and then the increase is rapid. A smoothed curve through the means of the columns will show that as ability in the Checker Puzzle increases the ability in Rational Learning slowly increases until the fifty-fifth percentile is reached and then there is a decrease in ability up to the seventy-fifth percentile.

Table XXXIV shows that the agreement between the scores in Rational Learning and the Tait Labyrinth Puzzle is not very

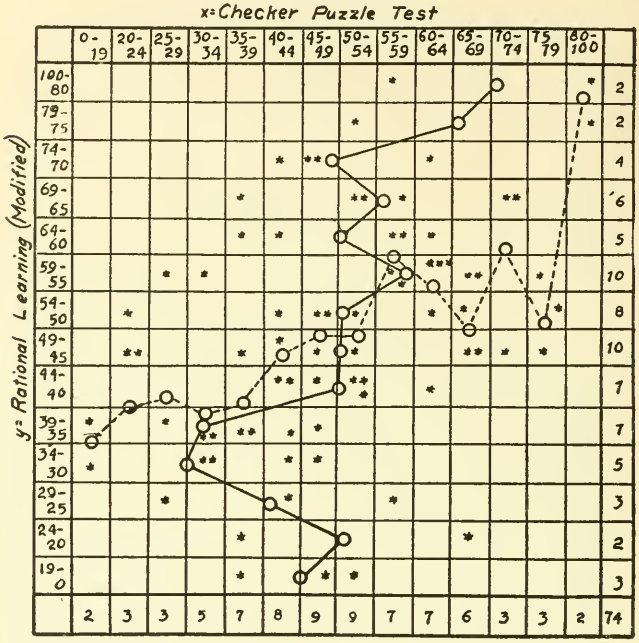
TABLE XXXIV. Showing the Distribution of Subjects on the Basis of Rational Learning and the Tait Labyrinth Puzzle. Circles through which the broken line passes represent the means of the columns and those through which the continuous line passes represent the means of the rows. Each asterisk represents a subject.



close. A smoothed curve through the means of the rows indicates that as ability in Rational Learning increases, there is a slight increase in ability to solve the Tait Labyrinth Puzzle up to the sixty-fifth percentile and then a slight decrease in ability from this point on. The line joining the means of the columns is very irregular. Beginning with the twenty-fifth percentile, as ability to solve the Tait Labyrinth Puzzle increases, there is a rapid increase in ability in Rational Learning. From this point on, there is little relation between the two sets of abilities.

Table XXXV shows the agreement between the scores in Rational Learning (Modified) and the Checker Puzzle. The line joining the means of the columns is linear according to the Blakeman test. It shows that as ability to solve the Checker Puzzle increases, there is also a constant but slow increase in

TABLE XXXV. Showing the Distribution of Subjects on the Basis of Rational Learning (Modified) and the Checker Puzzle. Circles through which the broken line passes represent the means of the columns and those through which the continuous line passes represent the means of the rows. Each asterisk represents a subject.



ability in Rational Learning (Modified). The line joining the means of the rows is non-linear. From the twentieth to the thirtieth percentiles, ability in the Checker Puzzle decreases with an increase in ability in Rational Learning (Modified). From the thirtieth percentile on, there is a slight increase in ability to solve the Checker Puzzle as the ability in Rational Learning (Modified) increases.

X. A Discussion of Learning and Intelligence

Learning of the reflective or problem solving kind has often been looked upon as involving, among other factors, one general mental function or process. It has been assumed that the person who has good reasoning ability in one problem will be good in all others of this same general type. This conception implies

that there is always a high correlation between any two such problems. In this investigation, however, the correlations are not high. In fact, they are all comparatively low. The lowest is .18 and the highest .44. The conclusion is that there is not one general rational learning process, but a number of processes. Two tests as similar as Rational Learning and Rational Learning (Modified), although they have something in common, are to a large degree independent of each other, since they have a correlation of only .36. This conception that any two tests have something in common and something that is not common has been explained in two ways. The first is the Two Factor theory of intelligence set forth by Spearman. The second assumes that each activity, such as a mental test or learning problem, involves a specific number of factors combined in a specific way. These two theories will now be treated in order.

Spearman set forth his Two Factor theory of intelligence in 1904. His first statement of the theory was as follows: "All branches of intellectual activity have in common one fundamental function (or group of functions), whereas the remaining or specific elements of the activity seem in every case to be wholly different from that in all the others."[33] This statement should be supplemented by the following explanation:

"It was never asserted, then, that the general factor prevails exclusively in the case of performances too alike: it was only said that when this likeness is diminished (or when the resembling performances are pooled together), a point is soon reached where the correlations are still of a considerable magnitude, but now indicate no common factor except the General one. The latter, it was urged, produces the basal correlation, while the similarities merely superpose something more or less adventitious."[34]

His most recent statement of the theory is:

"The purport of this theory is that the cognitive performances

[33] Spearman, C., General Intelligence, Objectively Determined and Measured, *Amer. J. Psychol.*, 1904, *15*, 201 ff.

[34] Hart, B., and Spearman, C., General Ability, Its Existence and Nature, *Brit. J. of Psychol*, 1912, *5*, 51 ff.

of any person depend upon: (a) A general factor entering more or less into them all; and (b) a specific factor not entering appreciably into any two, so long as these have a certain quite moderate degree of unlikeness to one another."[35]

Spearman's method was to measure a number of mental abilities in a number of persons and then calculate the correlation coefficients of each of these abilities with each of the others. He then noticed that these correlation coefficients had a certain relationship among themselves, which he called a hierarchial order. By this he means that if the coefficients of correlation of a number of mental functions are arranged in a descending order from left to right and from top to bottom as is usualy done in a correlation table, in every row the figures will be in a descending order as they are in the top row, and in every column the figures will be in a descending order as they are in the left column. This also means that in any table of correlations as ordinarily arranged, every column will have a perfect correlation with every other one. Spearman has also reduced this principle to the following exact mathematical equation:

$$r_{ap}/r_{aq} = r_{bp}/r_{bq},$$

in which a, b, p and q indicate any of the tests and r is the correlation between them.

It seems evident that the presence of such a general factor will always produce this hierarchy. In fact, if it can be shown that all correlations arrange themselves in such an order, it might be difficult to formulate any other theory to account for the facts. One exception, however, is enough to disprove the theory, since, if there is such a general factor, all correlations must take this hierarchical form. Thompson[36] has shown that it is possible with dice throws to get a set of correlation coefficients in excellent hierarchical order. He says that these imitation mental tests contain no general factor. Spearman, on the other hand, claims that Thompson let in a general factor at the

[35] Spearman, C., Manifold Sub-Theories of the "The Two Factors," *Psychol. Rev.*, 1920, *27*, 159 ff.

[36] Thompson, Godfrey H., General versus Group Factors in Mental Activities, *Psychol. Rev.*, 1920, *27*, 173 ff.

back door. It seems to the writer that Thompson has proved nothing more than that it is possible occasionally to get the hierarchial order of correlation coefficients when there is no general factor present. He has not weakened Spearman's argument in the least, provided Spearman can always get this order. Thompson further claims in this same article that the hierarchical order is the natural relationship among correlation coefficients. The writer is unable to see, however, just how his argument bears upon the question. He is willing, of course, to admit that this inability may be due to his lack of insight.

Spearman, in order to prove his theory, must show that every group of correlation coefficients of intellectual functions has, within the limits of experimental accuracy, this hierarchical order. Then his theory will hold only until it has been shown that this same order can be obtained consistently when there is no common factor present. The data of this investigation and their bearing upon the question are here presented.

TABLE XXXVI. Showing the Correlation Coefficients, when the Tests are Scored in the Light of the Criterion

	1	3	2	5	4
1		47	33	30	26
3	47		36	18	32
2	33	36		41	32
5	30	18	41		26
4	26	32	32	26	

In no column, except the first where it was deliberately arranged, does the hierarchy exist. Spearman would probably say that the mental functions here tested are too much alike for the criterion to hold. The correlations are so low, however, that this claim can hardly have weight. Our findings are indeed adverse to the Two Factor theory. If we examine the correlations and partial correlations in Table XXVII, it will be evident that no factor of any size whatever is common to all the mental

functions tested. For instance the correlation of tests 2 and 5 is .41. When the elements in test 3, common to 2 and 5, are removed, the correlation is still .38. This indicates that there is almost nothing common to the three tests. This same conclusion can be deduced from other cases in this same table. The correlation of tests 2 and 3 with the common elements in 5 removed, of tests 3 and 4 with the common elements in 5 removed, of tests 4 and 5 with the common elements in 3 removed, leads to the conclusion that there is no common factor large enough to account for all the correlations. Table XXXII shows the same conditions. The correlation of tests 2 and 3 with the common elements in 4 removed, of tests 2 and 5 with the common elements in 4 removed, of tests 3 and 4 with the common elements in 2 removed, shows that there are no elements common to all the tests sufficient to account for all the correlations.

Our data indicate that there is no common factor of any size running through all the tests. This amounts to saying that there is no such thing as general intelligence. What then is the nature of intelligence? One other theory will be considered. This theory assumes that in carrying out any activity, such as a mental test, a number of factors are at play. Each activity involves a specific number of factors combined in a specific way. The specific factors combined will differ with different individuals and with the same individual at different times. It will sometimes happen that a number of elements will run through several mental activities. In this case there may be said to be an element common to all the activities. In other cases there will be no element or elements common to more than two or three of the mental functions. For instance, tests 1 and 2 may correlate because of element a, tests 1 and 3 because of element b, tests 2 and 3 because of element c, etc.

This theory seems to be in harmony with the data of this investigation. According to the Two Factor theory, the correlations in Table XXXIII show that test 3, Rational Learning (Modified), must have more of the general factor than any other test; yet when the elements in this test common to the criterion

and test 2, to the criterion and test 4, or to the criterion and test 5, are removed, the correlation is still nearly three times the probable error. If test 3 contains more of the general factor than any other test and all correlation is due to this factor, then it should be reduced nearly to zero when the common elements in this factor are removed. On the other hand, the theory of various elements variously combined can easily account for all correlations and partial correlations. That is, tests 1 and 2 have common elements, some of which are found in each of the other three tests and some of which are not found in any of the other three tests. The same conclusion may be drawn from tests 1 and 3 and tests 1 and 5. Probably everything common to tests 1 and 4 is found in tests 2 and 3 or tests 3 and 5.

If we now turn to Table XXVII, it is evident from the viewpoint of the Two Factor theory that test 5 or test 2 has more of the general factor than any of the other tests; yet when the elements in 5 common to 3 and 4 are removed, the correlation is .28. This is an impossible result if the Two Factor theory is true. Table XXXII will also show similar conditions. Test 2 or test 5 must have enough of the general factor to make the correlation of these two tests .44. The other test may have more of this factor, but cannot have less if Spearman's theory is true. Now since the amount of the general factor involved in either of the two remaining tests, must be less than that involved in test 2 or test 5, the correlation of these tests, 3 and 4, should be reduced to zero, when the common elements in 2 or 5 are removed. Yet the correlation remains .31 when the common elements in both are removed.

These data, which cannot be explained at all by the Two Factor theory, are easily explained by the theory that intelligence consists of a large number of factors variously grouped and combined. Suppose that the correlation of tests 2 and 3 in table XXXII is due to elements a, b, c, d, e, f, g, h, i, and j. Now suppose that element a is the only one of these ten that is found in test 4, and that elements b, c, d, and e are the only ones of the ten found in test 5. When the element a is removed from the

ten common ones, the correlation is reduced from .33 to .29. In like manner, when the elements b, c, d, and e are removed, the correlation is reduced from .33 to .21. When the elements a, b, c, d, and e are removed, the correlation is reduced from .33 to .20. Thus, we have ten elements common to tests 2 and 3, one element common to tests 2, 3, and 4, four elements common to tests 2, 3, and 5. This analysis is not literally correct. There is undoubtedly a common factor of very small importance running through all four tests. This is evident from the fact that $r_{23\cdot45}$ is not much less than $r_{23\cdot4}$ or $r_{23\cdot5}$. That is, most of the elements in test 4 common to tests 2 and 3 are contained in the elements in test 5 common to tests 2 and 3. An examination of any of the correlations and the accompanying partial correlations will show that a very small factor runs through all four tests. This factor, however, is not sufficient to account for the correlations.

It seems that most of the investigations, when interpreted by Spearman, are in harmony with the Two Factor theory, but when interpreted by others, are adverse to this theory and more in harmony with the other theory here discussed. Thorndike[37] has recently made a study, using the Army Tests and a large number of subjects. His data are not in harmony with the Two Factor theory. He says in this article:

"We have considered the correlations obtained from time to time in various studies at Teachers College from the point of view of the Spearman theory, and have in general not been able to corroborate it. The most extensive data at our disposal (McCall, '16) seemed decidedly adverse."

Thorndike in this same article further says:

"We must, it appears, turn back with open mind to the details on intercorrelations and experimental analysis to work out the organization of intellect. Especially needed seem studies of the 'partial' inter-correlations with one after another of the factors equalized."

[37]Thorndike, Edward L., On the Organization of Intellect, *Psychol. Rev.*, 1921, *28*, 141 ff.

Simpson[38] discussed general intelligence and the bearings of his study upon the Two Factor theory. He says:

"We find justification for the common assumption that there is close inter-relation among certain mental abilities, and consequently a something that may be called 'general mental ability' or 'general intelligence'; and that on the other hand certain capacities are relatively specialized, and do not imply other abilities except to a very limited extent."

He says again: "We find no justification for the view that 'general intelligence' is to be explained on the basis of a hierarchy of mental functions, the amount of correlation in each case being due to the degree of connection with a common central factor."

Peterson[39] makes the following statement as to the nature of general intelligence: "General intelligence, if it is a reality at all, is probably not a separate constant factor, but a composite of many different abilities, and probably means different things in unlike situations, as different abilities are stressed. Such factors as energy and perseverance, degree of disturbance by emotions and self-consciousness, and many others that play their role in one's success in life, have not yet been successfully brought into the field of measurement by tests, especially by group tests."

XI. Summary and Conclusions

A. Method. The essential features of the method used in this investigation are:

(1) Four tests of the problem solving or rational learning type are used. Two of these tests have five kinds of data—time, repetitions, and three kinds of errors. One has three kinds of data—time, attempts and solutions. One has two kinds of data—time and number of trials. A criterion, the Stanford Revision of the Binet-Simon tests, is used in finding the best method of scor-

[38] Simpson, Benj. R., Correlations of Mental Abilities, *Teachers' College Contributions to Education*, 1912 (No. 53).

[39] Peterson, Joseph, Intelligence and Its Measurement, *J. of Educ. Psychol.*, 1921, *12*, 198 ff.

ing or combining the different kinds of data and in analyzing the tests.

(2) The raw scores in the criterion and in each factor of each test, and the final scores are transmuted into percentiles. This has been found very helpful in calculating correlations. For instance, all standard deviations as well as all the means are made approximately equal. The analysis of the curves through the means of the rows and columns is also simpler when the standard deviations are equal.

(3) Every factor in a test is correlated with the criterion and with every other factor. Then a complete set of partial correlations is worked out. This makes it possible to determine which factors must be used in scoring in order not to discard any elements in common with the criterion. For instance, it was found in Rational Learning that repetitions and perseverative errors contain everything in all the factors in common with the criterion. That is, everything in the other three factors is a duplication of these common elements in repetitions and perseverative errors.

(4) To determine the best combination of the factors that need to be retained, formula (2) is used. Formula (3) is then used to determine what this correlation is. As a check on the work, formula (1) is used. This gives the highest possible correlation of all the factors with the criterion. If this result agrees closely with that obtained from formula (2), it is evidence that the analysis and work are correct. It so happened that not more than two factors needed to be combined in any of the tests; but if it had been necessary to combine three or more factors, two would have been combined in the best way and then this result with the third factor. The writer has combined as many as five factors in this way and found it very satisfactory.

(5) For every correlation, Blakeman's criterion for linearity is applied. If non-linearity exists according to this criterion, the actual curves of the means of the rows and columns are constructed. The writer regards the correlation-ratio and the Blake-

man criterion of very little value unless the number of cases is large enough to eliminate most of the fluctuations in the curves through the means of the rows and columns. These fluctuations are often sufficient to produce a high correlation-ratio when there is no correlation, and, of course, non-linearity is indicated when the criterion is applied. The actual curves through the means of the rows and columns are of more significance, since it is possible to determine the general direction of such curves in making analyses.

(6) The final scores of each test are analyzed by comparing them with the criterion through the use of partial correlations and multiple correlation. It does not seem possible to do much with partial correlations in analysis unless a criterion is used.

(7) The final scores as obtained in the light of the criterion are correlated with each other and partial correlations worked out. In comparing the tests with each other, the multiple correlation method is found very valuable. Especially is this the case in determining how much each test has in common with all the others.

(8) As a final step in the technique, the tests are scored by combining all the factors in a test equally. This was thought best, since there was a possibility of accentuating certain elements in the tests by scoring them in the light of the criterion.

B. Results. The results indicated by the data are as follows:

(1) In scoring Rational Learning in the light of the criterion, repetitions and perseverative errors are the significant factors. Time, unclassified errors, and logical errors only duplicate the elements in these two factors. Time and unclassified errors are the significant factors in Rational Learning (Modified); the other three factors may be discarded. The number of solutions is the significant factor in the Checker Puzzle. Time and number of attempts add nothing. In the Tait Labyrinth Puzzle the number of trials is the significant factor. Time is unessential. It might be interesting to note here that in three of the four tests, time is an unessential factor. This does not mean, however, that the same results would have been obtained if the subject

had been told that time was not being considered. The correlations would probably have been very different. Time is probably not important in the Checker Puzzle and the Tait Labyrinth Puzzle. In Rational Learning the subject is controlled somewhat by the experimenter but in Rational Learning (Modified) he is free to go as fast as he wishes. This probably accounts for the difference in the value of the time factor in the two tests. In Rational Learning (Modified) the space perception makes it easier to avoid perseverative errors, and for that reason this factor becomes unessential. It is thought best to make no comparison of the difficulty of the two rational learning tests, since each subject had already taken Rational Learning (Modified), when he took Rational Learning. The similarity was usually recognized at once. It was not uncommon to have the subject say while he was reading the directions for Rational Learning, "This is just like that bell-ringing thing."

(2) Rational Learning and Rational Learning (Modified) seem to test or measure mental functions not detected by the intelligence tests. These may be summarized as follows: first, the ability to attack and solve a problem without getting confused; second, the ability to give attention longer than that usually required in mental tests; third, the type of attack made by the subject; and fourth, the speed of the subject. The objective data do not reveal any mental functions tested by the Checker Puzzle and the Tait Labyrinth Puzzle over and above those tested by the criterion.

(3) Rational Learning (Modified) correlates considerably higher with the criterion than does any of the other tests. This may be partly due to the fact that this test was given first. The Checker Puzzle has the lowest correlation with the criterion. Rational Learning (Modified) has elements in common with the criterion that are not found in the other three tests. The same is probably true of the Tait Labyrinth Puzle, but to a less extent. Rational Learning and the Checker Puzzle have nothing in common with the criterion that is not found in the other tests.

(4) When the tests are scored in the light of the criterion,

every correlation indicates something in common between the two tests correlated. The correlations of the second order also indicate that each pair of tests, except 3 and 5, have something in common that is not contained in the other tests. The factor running through all four tests is almost zero. Multiple correlation shows that each test has much that is not contained in the other three tests. The Checker Puzzle has most, the Tait Labyrinth comes second and Rational Learning has least. The differences are so small that they are probably not significant.

(5) When the tests are scored by combining all the factors in a test equally, the same general results are obtained as in the other method of scoring. There are some differences, however, in specific correlations. These are evident when tables XXVI and XXXII are examined. The correlations of test 2 with test 3 and test 5 are not changed much, but the correlation of test 2 with test 4 is reduced about half. The correlation of test 3 with test 4 is not changed much, but that of test 3 with test 5 is about doubled. The relations of test 4 with test 5 remain exactly the same. The multiple correlations show very little change in general by the two methods of scoring.

(6) There is nothing in the data of this investigation to justify the Two Factor theory of intelligence. In fact, everything is adverse to this theory. If the testing and the calculations are absolutely free from errors, the results obtained are impossible on the basis of the Two Factor theory. The correlations and partial correlations can be accounted for, however, by the theory that intelligence consists of various factors variously grouped for different situations.

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